MATHEMATICS

Study Material for JEE Main & Advanced preparation

Prepared by Career Point Kota Experts

CAREER POINT

CONTENTS OF THE PACKAGE AT A GLANCE

MATHEMATICS

Class 11

Trigonometry

- Trigonometric Ratios
- Trigonometrical Equations
- Properties of Triangle
- Radii of Circle
- Set & Relation
- Statistics

Algebra (Part-I)

- Elementary Mathematics & Logarithm
- Quadratic Equation
- Progressions
- Binomial Theorem
- Permutation & Combination

Coordinate Geometry

- Point & Straight Line
- Circle
- Parabola
- Ellipse
- Hyperbola

Class 12

Differential Calculus

- Function
- Inverse Trigonometric Functions
- Limit
- Continuity
- Differentiation
- Tangent & Normal
- Monotonicity
- Maxima & Minima

Integral Calculus

- Indefinite Integration
- Definite Integration
- Area Under the Curve
- Differential Equation

Algebra (Part-II)

- Complex Number
- Probability
- Determinants
- Matrices
- Vector
- Three Dimensional Geometry

Note to the Students

Career Point offers this must have Study Package in Physics to meet the complete curriculum needs of engineering aspirants. The set comprises of 6 books: **Mathematics** - set of 3 books for class 11 and set of 3 books for Class 12. The set caters to the different requirements of students in classes XI and XII. It offers complete and systematic coverage of **JEE Main** and **JEE Advanced** syllabi and aims to provide firm foundation in learning and develop competitive edge in preparation of the JEE and other engineering entrance examinations.

COMPONENTS OF EACH CHAPTER

These books are designed with an engaging and preparation-focused pedagogy and offer a perfect balance of conceptual learning and problem solving skills.

Theory & Concepts

Each chapter consists of high quality theory that covers all the topics, sub-topics and concepts of JEE syllabus.

Function

1. PRELIMINARIES

✦ Tricotomy Law

The real numbers are ordered in magnitude means. If x and y be two real numbers then there will be one and only one of the following relation will hold.

$x < y, x = y, x > y$

✦ Interval

The set of numbers between any two real numbers is called interval. The following are the types of interval.

(a) Closed Interval

$[a, b] = \{x, a \le x \le b\}$

(b) Open Interval

(a, b) or $]a, b[= \{x, a < x < b\}$

(c) Semi open or semi closed interval

$[a, b[$ or $[a, b) = \{x; a \le x < b\}$

$]a, b]$ or $(a, b] = \{x; a < x \le b\}$

2. DEFINITION OF FUNCTION

Let A and B be two non-empty sets. Then a function ' f ' from set A to set B is a rule which associates elements of set A to elements of set B such that

- All elements of set A are associated to element in set B.
- An element of set A is associated to a unique element in set B.

Terms such as "map" (or "mapping"), "correspondence" are used as synonyms for "function". If f is a function from a set A to set B, then we write $f : A \to B$ or $A \xrightarrow{f} B$, which is read as f is a function from A to B or f maps A to B.

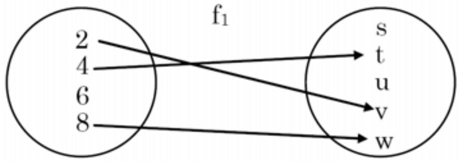
✦ Pre Image / f Image

If an element $a \in A$ is associated to an element $b \in B$, then b is called 'the f-image of a' or 'image of a under f' or 'the value of the function f at a'. Also, a is called the pre-image of b under the function f. We write it as : $b = f(a)$.

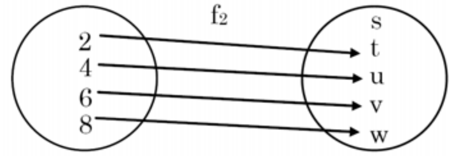
Example. 1

Let $A = \{2, 4, 6, 8\}$ and $B = \{s, t, u, v, w\}$ be two sets and let f_1, f_2, f_3 and f_4 be rules associating elements of A to elements of B as shown in the following figures.

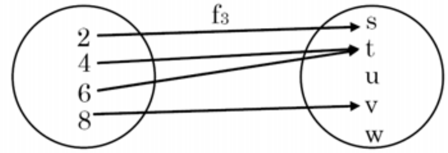
f_1



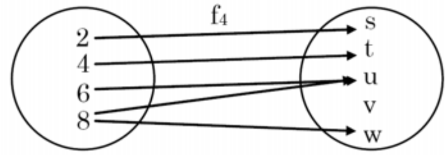
f_2



f_3



f_4



Now see that f_1 is not function from set A to set B, since there is an element $6 \in A$ which is not associated to any element of B. But f_2 and f_3 are the functions from A to B, because under f_2 and f_3 each element of A is associated to a unique element in B. But f_4 is not a function from A to B because an element $8 \in A$ is associated to two element u and w in B.

3. WAYS OF REPRESENTING FUNCTIONS

✦ Analytical Representation

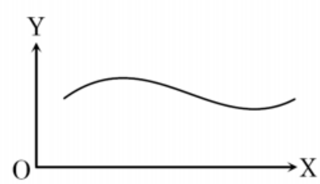
$y = \sqrt{x^2 - 1}$, $f(x) = \dfrac{\log_e x + e^x}{\sin x}$

$f(x) = \dfrac{ax^2 + bx + c}{e^{2x} \sin^{-1} x}$, ...

Representation of a function in above way is called analytical representation. i.e. when function is denoted like $y = f(x)$ or $f(x, y) = 0$, then it is called **Analytical Representation**.

✦ Graphical Representation

In 2D a set of points M(x, y) provided no two or more points lie in same straight line parallel to axis of y. Then M(x, y) represents a function, where x's denotes arguments and y denotes the value of function.



Important Points

This part contains important concepts & formulas of chapter at one place in short manner, So that student can revise all these in short time.

Points to Remember

The function whose period is 2π

1. $(\sin x)^{2n+1}, (\cos x)^{2n+1}, (\sec x)^{2n+1}, (\text{cosec } x)^{2n+1}$

The function whose period is π

2. $(\sin x)^{2n}$, $(\cos x)^{2n}$, $(\sec x)^{2n}$, $(\text{cosec } x)^{2n}$

3. $(\tan x)^n$, $(\cot x)^n$

4. $|\sin x|$, $|\cos x|$, $|\tan x|$, $|\cot x|$, $|\sec x|$, $|\text{cosec } x|$

5. If f(x) has the period T, then f(± ax + b) will have the period $\frac{T}{|a|}$

6. If $f_1(x)$ has the period T_1

$f_2(x)$ has the period T_2

Then period of $af_1(x) + bf_2(x)$ will be

Solved Examples (JEE Main / Advanced)

To understand the application of concepts, there is a solved example section. It contains large variety of all types of solved examples with explaination to ensure understanding the application of concepts.

SOLVED EXAMPLES

Ex.1 Find the domain and range of the function $f(x) = \sqrt{2-x} + \sqrt{1+x}$

Sol. Domain of $f(x) = \{x \mid 2 - x \geq 0 \text{ and } 1 + x \geq 0\}$

$\therefore$ domain of $f(x) = [-1, 2]$

Again, $\{f(x)\}^2 = \left(\sqrt{2-x} + \sqrt{1+x}\right)^2$

$= 3 + 2\sqrt{(2-x)(1+x)}$

$= 3 + 2\sqrt{2+x-x^2}$

$= 3 + 2\sqrt{\frac{9}{4} - \left(x - \frac{1}{2}\right)^2}$

$\therefore$ the greatest value of $\{f(x)\}^2$

$= 3 + 2.\sqrt{\frac{9}{4}} = 6$, when $x = \frac{1}{2}$

the least value of $\{f(x)\}^2 = 3 + 0 = 3$,

when $x - \frac{1}{2} = \frac{3}{2}$, i.e. $x = 2$

$\therefore$ the greatest value of $f(x) = \sqrt{6}$

and the least value of $f(x) = \sqrt{3}$

$\therefore$ range of $f(x) = [\sqrt{3}, \sqrt{6}]$

Ex.2 Find the range of the following function

$$f(x) = \frac{3}{2-x^2}$$

$= \log_2\left(\sin\left(\pi - \frac{\pi}{4}\right) + 3\right) = y$ (let)

$\Rightarrow \quad 2^y = \sin\left(\pi - \frac{\pi}{4}\right) + 3$

$\Rightarrow \quad 2^y - 3 = \sin\left(\pi - \frac{\pi}{4}\right)$

But $-1 \leq \sin\left(\pi - \frac{\pi}{4}\right) \leq 1$

$\therefore \quad -1 \leq 2^y - 3 \leq 1$

$\Rightarrow \quad 2 \leq 2^y \leq 4$

$\Rightarrow \quad 2^1 \leq 2^y \leq 2^2$

Hence $y \in [1, 2]$.

Hence Range of $f(x)$ is [1, 2].

Ex.4 Find the period of the following function

$f(x) = e^{x-[x] + |\cos \pi x| + |\cos 2\pi x| + ... + |\cos n\pi x|}$,

[.] is greatest integer function.

Sol. $f(x) = e^{x-[x] + |\cos \pi x| + |\cos 2\pi x| + ... + |\cos n\pi x|}$

Period of $x - [x] = 1$

Period of $|\cos \pi x| = 1$

Period of $|\cos 2\pi x| = 1/2$

....

....

Period of $|\cos n\pi x| = 1/n$

So period of $f(x)$ will be

Practice Exercises

Exercise Level - 1 : It contains objective questions with single correct choice to ensure sufficient practice to accutrately appply formule and concepts.

Exercise Level - 2 : It contains single objective type questions with moderate difficulty level to enhance the conceptual and application level of the student.

Exercise Level - 3 : It contains all variety of questions as per level of JEE Advanced such as MCQ, Column match, Passage based & Numerical type etc.

EXERCISE (Level-3)

Part-A : Multiple correct answer type questions

Q.1 If $f(x) = \sqrt{x^2 - |x|}$, $g(x) = \frac{1}{\sqrt{9-x^2}}$ then D_{f+g} contains
(A) $(-3, -1)$ (B) $[1, 3)$
(C) $[-3, 3]$ (D) $\{0\} \cup [1, 3)$

Q.2 If $f(x) = \frac{3x-1}{3x^3+2x^2-x}$ and $S = \{x \mid f(x) > 0\}$ then S contains
(A) $(-\infty, -2)$ (B) $\left(\frac{1}{3}, 5\right)$
(C) $(-\infty, -1)$ (D) $(0, \infty) - \left\{\frac{1}{3}\right\}$

Q.3 If D is the domain of the function
$f(x) = \sqrt{1-2x} + 3\sin^{-1}\left(\frac{3x-1}{2}\right)$ then D contains-
(A) $\left[-\frac{1}{3}, \frac{1}{2}\right]$ (B) $\left[-\frac{1}{3}, 0\right]$
(C) $\left[-\frac{1}{3}, 1\right]$ (D) $\left[\frac{1}{2}, 1\right]$

Q.4 Let $A = R - \{2\}$ and $B = R - \{1\}$.
Let $f : A \to B$ be defined by $f(x) = \frac{x-3}{x-2}$ then–
(A) f is one-one (B) f is onto
(C) f is bijective (D) None of these

Q.5 If $F(x) = \frac{\sin \pi[x]}{\{x\}}$, then $F(x)$ is:
(A) Periodic with fundamental period 1
(B) Even
(C) Range is singleton
(D) Identical to $\operatorname{sgn}\left(\operatorname{sgn}\frac{\{x\}}{\sqrt{\{x\}}}\right) - 1$, where $\{x\}$ denotes fractional part function and [.] denotes greatest integer function and sgn (x) is a signum function.

Q.6 Let $f : [-1, 1] \to [0, 2]$ be a linear function which is onto then $f(x)$ is/are
(A) $1 - x$ (B) $1 + x$ (C) $x - 1$ (D) $x + 2$

Q.7 In the following functions defined from $[-1, 1]$ to $[-1, 1]$ the functions which are not bijective are:
(A) $\sin(\sin^{-1}x)$ (B) $\frac{2}{\pi}\sin^{-1}(\sin x)$
(C) $(\operatorname{sgn} x)\, \ell n\, e^x$ (D) $x^3 \operatorname{sgn} x$

Q.8 Which of the following function is periodic ?
(A) $\operatorname{sgn}(e^{-x})$
(B) $\sin x + |\sin x|$
(C) $\min(\sin x, |x|)$
(D) $\left[x + \frac{1}{2}\right] + \left[x - \frac{1}{2}\right] + 2[-x]$
Where $[x]$ denotes greatest integer function.

Q.9 If $f(x) = \begin{cases} 2x+3 & x \le 1 \\ a^2x+1 & x > 1 \end{cases}$ then values of 'a' for which $f(x)$ is injective is
(A) –3 (B) 3 (C) 0 (D) 1

Q.10 Consider the function $y = f(x)$ satisfying the condition $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ $(x \neq 0)$, then
(A) domain of $f(x)$ is R
(B) domain of $f(x)$ is R – (–2, 2)
(C) range of $f(x)$ is $[-2, \infty)$
(D) range of $f(x)$ is $[2, \infty)$

Q.11 Consider the real-valued function satisfying $2f(\sin x) + f(\cos x) = x$. Then
(A) domain of $f(x)$ is R
(B) domain of $f(x)$ is $[-1, 1]$
(C) range of $f(x)$ is $\left[-\frac{2\pi}{3}, \frac{\pi}{3}\right]$
(D) range of $f(x)$ is R

Q.12 Let $f(x) = x^2 - 2ax + a(a + 1)$, $f : [a, \infty) \to [a, \infty)$. If one of the solutions of the equation $f(x) = f^{-1}(x)$ is 5049, then the other may be
(A) 5051 (B) 5048 (C) 5052 (D) 5050

Q.13 If $f : R^+ \to R^+$ is a polynomial function satisfying the functional equation $f(f(x)) = 6x - f(x)$, then $f(17)$ is equal to -
(A) 17 (B) –51 (C) 34 (D) –34

Q.14 $f : R \to [-1, \infty)$ and $f(x) = \ell n([|\sin 2x| + |\cos 2x|])$ (where [.] is the greatest integer function)
(A) $f(x)$ has range Z
(B) $f(x)$ is periodic with fundamental period $\pi/4$
(C) $f(x)$ is invertible in $\left[0, \frac{\pi}{4}\right]$
(D) $f(x)$ is into function

Q.15 Let $f(x) = \operatorname{sgn}(\cot^{-1}x) + \tan\left(\frac{\pi}{2}[x]\right)$, where $[x]$ is the greatest integer function less than or equal x. Then which of the following alternatives is/are true ?
(A) $f(x)$ is many one but not even function
(B) $f(x)$ is periodic function
(C) $f(x)$ is bounded function
(D) Graph of $f(x)$ remains above the x-axis

Exercise Level - 4 : It contains previous years question of JEE Main (Section-A)/Advanced (Section-B) from Year 2005 to 2023.

EXERCISE (Level-4)

Old Examination Questions

Section-A [JEE Main]

Q.1 Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\dfrac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval **[AIEEE-2005]**

(A) $\left(0, \dfrac{\pi}{2}\right)$ (B) $\left[0, \dfrac{\pi}{2}\right)$

(C) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ (D) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

Q.2 A real valued function $f(x)$ satisfies the functional equation $f(x - y) = f(x) f(y) - f(a - x) f(a + y)$ where a is a given constant and $f(0) = 1$, then $f(2a - x)$ is equal to - **[AIEEE-2005]**

(A) $-f(x)$ (B) $f(x)$

(C) $f(a) + f(a - x)$ (D) $f(-x)$

Q.3 The largest interval lying in $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$ for which the function

$f(x) = 4^{-x^2} + \cos^{-1}\left(\dfrac{x}{2} - 1\right) + \log(\cos x)$ defined, is- **[AIEEE 2007]**

(A) $[0, \pi]$ (B) $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$

(C) $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ (D) $\left[0, \dfrac{\pi}{2}\right)$

Q.4 Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in N : y = 4x + 3$ for some $x \in N\}$. Inverse of f is - **[AIEEE 2008]**

(A) $g(y) = 4 + \dfrac{y+3}{4}$ (B) $g(y) = \dfrac{y+3}{4}$

(C) $g(y) = \dfrac{y-3}{4}$ (D) $g(y) = \dfrac{3y+4}{3}$

Q.5 For real x, let $f(x) = x^3 + 5x + 1$, then

(A) Statement -1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement -1

(B) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement -1.

(C) Statement -1 is true, Statement-2 is false.

(D) Statement -1 is false, Statement-2 is true

Q.7 The domain of the function

$f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is : **[AIEEE 2011]**

(A) $(-\infty, \infty)$ (B) $(0, \infty)$

(C) $(-\infty, 0)$ (D) $(-\infty, \infty) - \{0\}$

Q.8 Let A and B be nonempty set in R and $f : A \to B$ be a bijective function.

Statement-1: f is an onto function

Statement-2 : There exists a function $g : B \to A$ such that $fog = I_B$.

[AIEEE Online- 2012]

(A) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1

(B) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of statement-1

(C) Statement-1 is true, Statement-2 is false

(D) Statement-1 is false, Statement-2 is true

Q.9 The range of the function $f(x) = \dfrac{x}{1+|x|}$, $x \in R$ is : **[AIEEE Online- 2012]**

(A) $[-1, 1]$ (B) R (C) $R - \{0\}$ (D) $(-1, 1)$

Q.10 If P(S) denotes the set of al subsets of a given set S, then the number of one to one functions from the set S = {1, 2, 3} to the set P(S) is : **[AIEEE Online- 2012]**

(A) 24 (B) 8 (C) 336 (D) 320

Q.11 Let A = {1, 2, 3, 4} and R : A → A be the relation defined by R = {(1, 1), (2, 3), (3, 4), (4, 2)}. The correct statements is : **[JEE Main Online -2013]**

Exercise Level - 5 : Advanced level a bit complex questions for students for solid rock prepertion for Top Rankers.

Answer key

Ankswer key is provided at the end of the exercise sheets.

ANSWER KEY

EXERCISE (Level-1)

1. (A)	**2.** (D)	**3.** (C)	**4.** (D)	**5.** (A)	**6.** (C)	**7.** (B)
8. (C)	**9.** (C)	**10.** (B)	**11.** (C)	**12.** (A)	**13.** (D)	**14.** (C)
15. (A)	**16.** (D)	**17.** (B)	**18.** (D)	**19.** (B)	**20.** (A)	**21.** (B)
22. (C)	**23.** (B)	**24.** (C)	**25.** (A)	**26.** (A)	**27.** (A)	**28.** (B)
29. (B)	**30.** (B)	**31.** (A)	**32.** (D)	**33.** (B)	**34.** (D)	**35.** (A)
36. (A)	**37.** (A)	**38.** (D)	**39.** (C)	**40.** (A)	**41.** (A)	

Revision Plan

We emphasis that every student should prepare his/her own revision plan. For this purpose there is Revision Plan Section in each chapter which student should prepare while going thorugh the study material. This will be useful at the time of final revision before final exam for quick & effective revision.

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

A. Write Question Number (QN) which you are unable to solve at your own in **column A**.

B. After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.

C. Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Level-1		
Level-2		
Level-3		
Level-4		
Level-5		

Online Solutions

Self explanatory and detailed soltuion of all excercises above are available on Career Point website **www.careerpoint.ac.in**

FUNCTION

EXERCISE (Level-1)

Answer Key & Solution

Question Number	Solution
1	Click Here
2	Click Here
3	Click Here
4	Click Here
5	Click Here
6	Click Here
7	Click Here
8	Click Here
9	Click Here
10	Click Here
11	Click Here
12	Click Here
13	Click Here
14	Click Here
15	Click Here
16	Click Here
17	Click Here
18	Click Here
19	Click Here
20	Click Here
21	Click Here

Question Number	Solution
22	Click Here
23	Click Here
24	Click Here
25	Click Here
26	Click Here
27	Click Here
28	Click Here
29	Click Here
30	Click Here
31	Click Here
32	Click Here
33	Click Here
34	Click Here
35	Click Here
36	Click Here
37	Click Here
38	Click Here
39	Click Here
40	Click Here
41	Click Here

FUNCTION

JEE ADVANCED SYLLABUS

1. *Domain and range of functions*

2. *Into, Onto and one-to-one function*

3. *Sum, Difference, Product and quotient of two functions*

4. *Composite Function*

5. *Absolute value*

6. *Greatest integer, Polynomial, Rational, Trigonometric, Exponential and logarithmic functions*

7. *Even and odd functions*

8. *Inverse of a function*

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

A. Write Question Number (QN) which you are unable to solve at your own in **column A**.

B. After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.

C. Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Level-1		
Level-2		
Level-3		
Level-4		
Level-5		

Revision Strategy:

Whenever you wish to revision this chapter, follow the following steps-

Step-1: Review your theory notes.

Step-2: Solve Questions of Column A

Step-3: Solve Questions of Column B

Step-4: Solve questions from other Question Bank, Problem book etc.

1. PRELIMINARIES

✦ Tricotomy Law

The real numbers are ordered in magnitude means. If x and y be two real numbers then there will be one and only one of the following relation will hold.

$x < y, x = y, x > y$

✦ Interval

The set of numbers between any two real numbers is called interval. The following are the types of interval.

(a) Closed Interval

$[a, b] = \{x, a \le x \le b\}$

(b) Open Interval

(a, b) or $]a, b[= \{x, a < x < b\}$

(c) Semi open or semi closed interval

$[a, b[$ or $[a, b) = \{x; a \le x < b\}$

$]a, b]$ or $(a, b] = \{x; a < x \le b\}$

2. DEFINITION OF FUNCTION

Let A and B be two non-empty sets. Then a function ' f ' from set A to set B is a rule which associates elements of set A to elements of set B such that

- All elements of set A are associated to element in set B.
- An element of set A is associated to a unique element in set B.

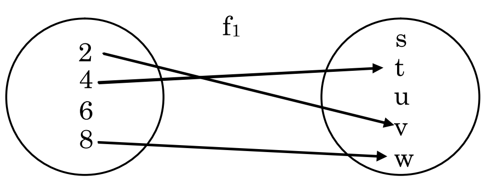
Terms such as "map" (or "mapping"), "correspondence" are used as synonyms for "function". If f is a function from a set A to set B, then we write $f : A \to B$ or $A \xrightarrow{f} B$, which is read as f is a function from A to B or f maps A to B.

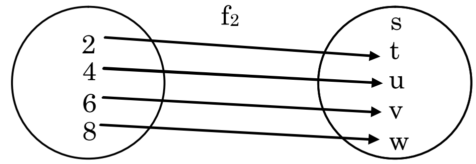
✦ Pre Image / f Image

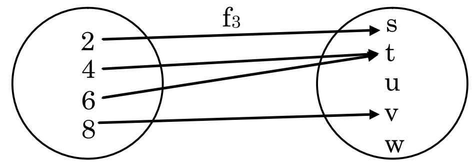
If an element $a \in A$ is associated to an element $b \in B$, then b is called 'the f-image of a' or 'image of a under f' or 'the value of the function f at a'. Also, a is called the pre-image of b under the function f. We write it as : $b = f(a)$.

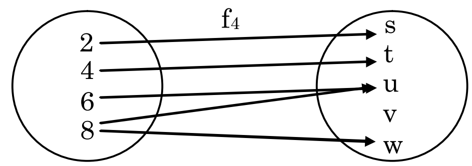
Example. 1

Let A = {2, 4, 6, 8} and B = {s, t, u, v, w} be two sets and let f_1, f_2, f_3 and f_4 be rules associating elements of A to elements of B as shown in the following figures.









Now see that f_1 is not function from set A to set B, since there is an element $6 \in A$ which is not associated to any element of B. But f_2 and f_3 are the functions from A to B, because under f_2 and f_3 each element of A is associated to a unique element in B. But f_4 is not a function from A to B because an element $8 \in A$ is associated to two element u and w in B.

3. WAYS OF REPRESENTING FUNCTIONS

✦ Analytical Representation

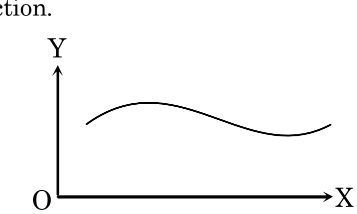
$y = \sqrt{x^2 - 1}$, $f(x) = \dfrac{\log_e x + e^x}{\sin x}$

$f(x) = \dfrac{ax^2 + bx + c}{e^{2x} \sin^{-1} x}$, ...

Representation of a function in above way is called analytical representation. i.e. when function is denoted like $y = f(x)$ or $f(x, y) = 0$, then it is called **Analytical Representation**.

✦ Graphical Representation

In 2D a set of points M(x, y) provided no two or more points lie in same straight line parallel to axis of y. Then M(x, y) represents a function, where x's denotes arguments and y denotes the value of function.



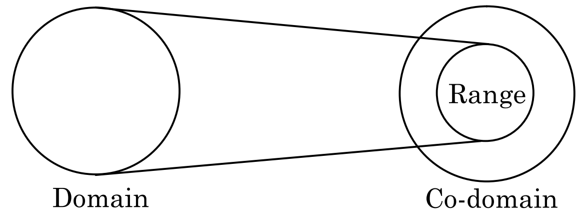
✦ Mapping

A mapping $f : X \to Y$ is said to be function if each element in the set X has it's image in set Y. This may be possible that the set Y may contain same such elements which may not be the images of any element of set X.

Each element in set X can not have more than one image. But this is possible that more than one element of X can have the same image.

- **Domain, Co-domain :** Set X is called domain of f i.e. Set of those elements from which functions is to be define and set Y is called Co-domain of f. i.e. Set of those elements into which the function is to be define.
- **Range of f :** Set of images of each element of X, is called range of f.

NOTE **Range $\subseteq$ Co domain**



REMEMBER

Dom $(f + g + h....)$ = Dom f $\cap$ Dom g $\cap$ Dom h....
Dom $(f - g)$ = Dom f $\cap$ Dom g
Dom $(f \times g \times h ...)$ = Dom f $\cap$ Dom g $\cap$ Dom h....
Dom (f/g) = Dom f $\cap$ Dom g $- \{x : g(x) = 0\}$

✦ Function as an ordered pair

Let A and B be two non - empty sets. A relation from A to B, i.e., a sub -set of $A \times B$, is called a function (or a mapping or a map) from A to B if

- For each $a \in A$ there exists $b \in B$ such that $(a, b) \in f$,
- $(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$.

Thus, a non - empty subset f of $A \times B$ is a function from A to B if each element of A appear in some ordered pair in f and no two ordered pairs in f have the same first element.

If $(a, b) \in f$, then b is called the image of a under f.

Example. 2

Let A = {a, b, c }, B = {2, 3, 4} and f_1 , f_2 and f_3 .... be subsets of $A \times B$ then

$f_1 = \{(a, 2), (b, 3), (c, 4)\}$
$f_2 = \{(a, 2), (a, 3), (b, 3), (c, 4)\}$,
$f_3 = \{(a, 3), (a, 4)\}$
$f_4 = \{(a, 2), (c, 3), (b, 4)\}$
....

Then f_1 and f_4 is a function from A to B but f_2 and f_3 are not functions from A to B.

Reason :

The element in f_2 has two images as 2 and 3 also in f_3 also is the same that is why f_2 and f_3 are not the functions.

4. DOMAIN

Set of those values of x for which f(x) is defined called domain of y = f(x). For example $y = \log_e x$ is defined for $x > 0$ therefore domain of $y = \log_e x$ is R^+. y = sin x is defined $\forall$ x therefore domain of sin x is R.

Example Based on

Domain of Definition

Example. 3

Find the domain of $f(x) = \log_{10} \sin x$

Solution.

For existence of f(x), sin x > 0 and from the graph of y = sin x it is clear that sin x is +ve when $2n\pi < x < (2n + 1)\pi$

Hence Dom f(x) = $(2n\pi, (2n + 1)\pi)$, where $n \in I$.

Example. 4

Find the domain of definition of $f(x) = \dfrac{1}{\sqrt{x - |x|}}$

Solution.

$x - |x| > 0$
$\Rightarrow$ $x > |x|$
and already we know $|x| \geq x$.
this contradiction
$\Rightarrow$ Dom f(x) = ϕ.

Example. 5

Find the domain of $f(x) = \sin x + \cos x + e^x \tan x$.

Solution.

Dom sin x = R
Dom cos x = R
Dom tan x = $R - \left\{\dfrac{2n+1}{2}\pi\right\}$

Dom e^x = R

$\therefore$ Dom f = Dom sin x $\cap$ Dom cos x $\cap$ Dom e^x tan x

$$= R \cap R \cap R - \left\{\frac{2n+1}{2}\pi\right\} = R - \left\{\frac{2n+1}{2}\pi\right\}$$

5. RANGE

Set of values of f(x) which it attains at points in its domain is called as range of f(x).

Example Based on

Range of Function

Example. 6

Find the range of function $f(x) = \sqrt{\sin x}$

Solution.

$-1 \le \sin x \le 1 \quad \Rightarrow 0 \le \sqrt{\sin x} \le 1$

$\therefore$ Range f = [0, 1]

Example. 7

Find the range of $f(x) = 3^{\sin^{-1} x}$

Solution.

$-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$

$3^{-\pi/2} \le \ \le 3^{\pi/2}$

$\Rightarrow \quad 3^{-\pi/2} \le f(x) \le 3^{\pi/2}$

$\therefore$ Range f = $[3^{-\pi/2}, 3^{\pi/2}]$

6. KINDS OF FUNCTION

The following are the kinds of functions :

✦ One-One Function (Injective)

If each element in the domain of a function has a distinct image in the co-domain, the function is said to be One-One. One-One function is also known as Injective Function.

e.g. $f : R \to R^+$ given by $y = e^x$ $y = 2^{-x}$,
$g : R \to R$ given & $g(x) = 3x - 7$ are One-One functions.

or, $f : A \to B$ is one-one

$\Leftrightarrow$ $a \ne b \Rightarrow f(a) \ne f(b)$ for all $a, b \in A$

$\Leftrightarrow$ $f(a) = f(b) \Rightarrow a = b$ for all $a, b \in A$

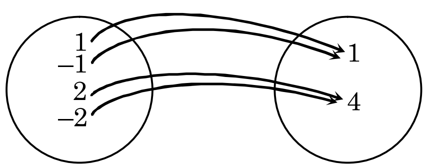
e.g. $y = \sin^{-1} x$, $y = \cos^{-1}x$,
$y = e^x$, $y = \log_e x$, ...

✦ Many-One Function

If two or more than two elements of domain have the same image. Then f(x) is called Many-One.

e.g. $f : R \to R^+$; $f(x) = x^2 + 4$
$g : R \to R^+$; $g(x) = x^8 + x^4 + x^2 + 4$
Many one

eg $f(x) = x^2$



- **Horizontal line Test :**
  If the graph of y = f(x) is given and the line parallel to x - axis cuts the curve at more than one point then function is many-one.
  or, $f : A \to B$ is a many - one function if there exist $x, y \in A$ such that $x \ne y$ but $f(x) = f(y)$.
  e.g $y = \sin x$, $y = \cos x$, $y = \tan x$, $y = x^2$, $y = x^4$, ...

✦ Onto Function (Surjective)

Let, $f : X \to Y$ be a function. If each element in the co-domain Y has at least one pre-image in the domain X. i.e.

Range f = Co domain

Then f is called **Onto**.

Onto function are also called surjective and if function be both One-One and Onto then function is called **Bijective**.

or, $f : A \to B$ is a surjection iff for each $b \in B$, $\exists\, a \in A$ such that $f(a) = b$.

e.g. Let $f : R^+ \to R$ is defined by $y = \log_2 x$, then f(x) is Onto function.

But when $f : R \to R$ is defined by $f(x) = \sin x$, then f(x) is not Onto function.

✦ Into Function

If there exist one or more than one element in the Co-domain Y which is not an image of any element in the domain X. Then f is Into.

In other words $f : A \to B$ is an into function if it is not an onto function.

e.g. Let $f : R \to R$ is defined by $y = x^2 + 1$, then f(x) is an Into function.

But when $f : R \to [0,\infty)$ is defined by $y = x^2$, then f(x) is not Into function. (**Note it**)

REMEMBER

(i) If $\frac{dy}{dx} > 0$, $\forall$ x in domain then f is One One.

(ii) If $\frac{dy}{dx} < 0$, $\forall$ x in domain then f is One-One.

(iii) If a continuous function f(x) which has either local minima or maxima or both then f(x) will be Many-One.

(iv) Every even function is Many-one.

(v) Every periodic function is Many-one.

7. WHEN THE FUNCTION IS CALLED 'DEFINED' OR 'NOT DEFINED'

If

(i) f(x) gives some imaginary value at some point.

(ii) f(x) gives set of imaginary values in an interval.

(iii) f(x) is indeterminate form like $\frac{0}{0}, \frac{\infty}{\infty}$, ...

Then f(x) is said to be not defined or undefined.

If

(i) f(x) is real and unique at some point (say x = a)

(ii) f(x) is real and unique for corresponding values in an interval.

Then f(x) is said to be defined.

Consider $f(x) = \frac{x^2 - a^2}{x - a}$, obviously f(x) is not defined at x = a, because $f(a) = \frac{0}{0}$.

At other point is well defined because f(x) is real and unique other than x = a i.e. for $x \ne a$.

Again Let, $f(x) = \frac{1}{x-a}$, At x = a, f(x) = ∞. Therefore f(x) is not defined at x = a and defined for x ≠ a because f(x) is real for x.

Let $f(x) = \sqrt{x}$,

Here f(x) gives imaginary values for x < 0.

Therefore f(x) is not defined for x < 0 and f(x) is defined for x ≥ 0.

Let, $f(x) = \log_e x$

For x < 0 ; f(x) is imaginary

For x = 0 ; f(x) = – ∞

For x > 0 ; f (x) is real

Here f(x) is not defined for x ≤ 0 and defined for x > 0

Example Based on

When the Function is Called 'Defined' or 'Not Defined'

Example. 8

Let $f(x) = 3^{\cos^{-1}\left(\log_e \sqrt{1-e^{2x}}\right)}$ where f(x) is not defined

Solution.

$\log_e \sqrt{1-e^{2x}} > 1$, or $\log_e \sqrt{1-e^{2x}} < -1$

Also $1 - e^{2x} > 0$ ⇒ x < 0, ...(i)

⇒ $\sqrt{1-e^{2x}} > e$, or $\sqrt{1-e^{2x}} < e^{-1}$

⇒ $1 - e^{2x} > e^2$, or $1 - e^{2x} < e^{-2}$

⇒ $-e^{2x} > e^2 - 1$, or $-e^{2x} < e^{-2} - 1$

⇒ $e^{2x} < 1 - e^2$, or $e^{2x} > 1 - e^{-2}$

⇒ but $e^{2x} > 0$,

⇒ $x > \frac{1}{2}(\log 1 - e^{-2})$ ...(ii)

From (i) and (ii), f(x) is defined for

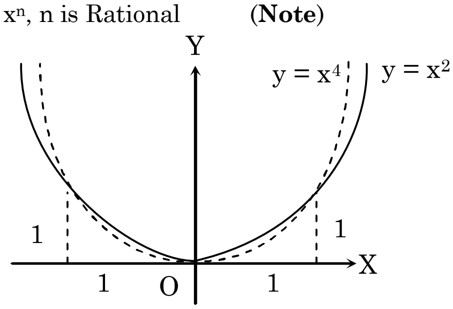
$x \in (1/2 \log_e (1 - e^{-2}), 0)$

8. BASIC ELEMENTARY FUNCTIONS

The basic elementary function are the following functions with analytic representation.

✦ Power Function

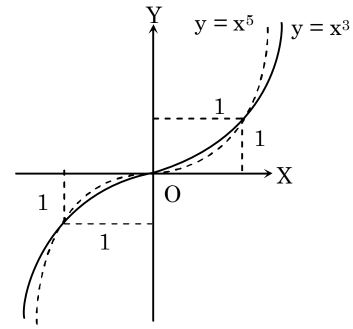
$y = x^n$, n is Rational (**Note**)



Domain : R

Range : $R^+ \cup \{0\}$

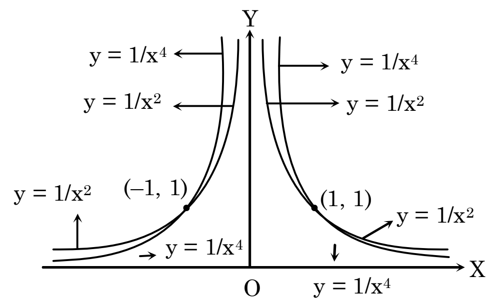
Nature : Many one into



Domain : R

Range : R

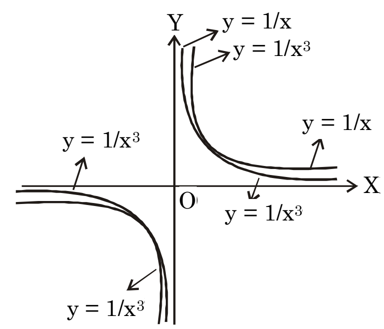
Nature : one one onto



Domain : R – {0}

Range : R^+

Nature : Many one into



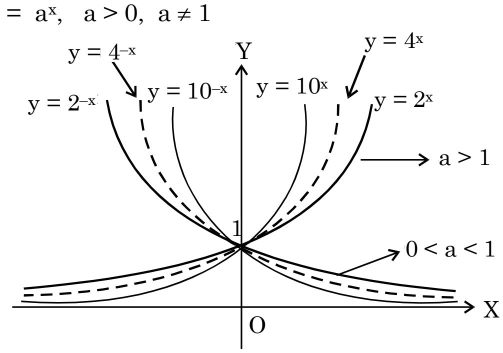
Domain : R – {0}

Range : R – {0}

Nature : one - one, into

✦ General Exponential Function

$y = a^x$, a > 0, a ≠ 1



Domain : R

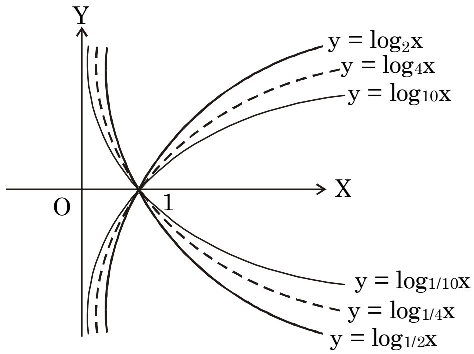
Range : R^+

Nature : one one into

NOTE **For a < 0, The graph of function is not defined.**

✦ Logarithmic Function

$y = \log_a x, \quad a > 0, \quad a \neq 1$



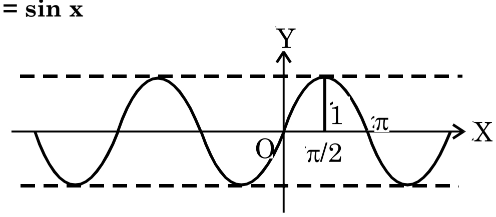
Domain : R^+
Range : R
Nature : one one into

NOTE **As the base increases curve is more near to both the axis.**

✦ Trigonometric Function or Circular Function

y = sin x, y = cos x, y = tan x
y = sec x, y = cot x, y = cosec x

y = sin x

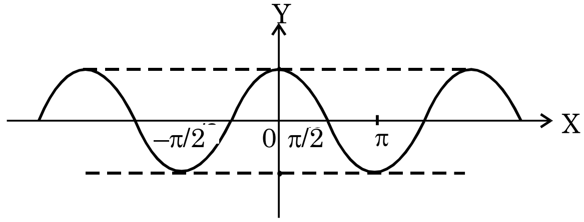


Domain : R
Range : [–1, 1]
Nature : Many one into
Principle value of x : [–π/2, π/2]

NOTE **The graph of y = sin x is symmetric about origin i.e. symmetric in opposite quadrants.**

Reason :
sin x is odd function and every odd function is symmetric about origin.

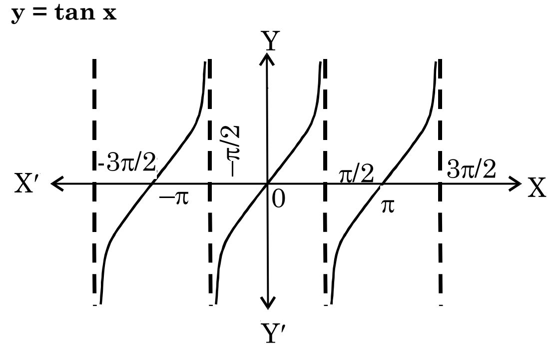
y = cos x



Domain : R
Range : [–1, 1]
Nature : Many one into
Principle value of x : [0, π]

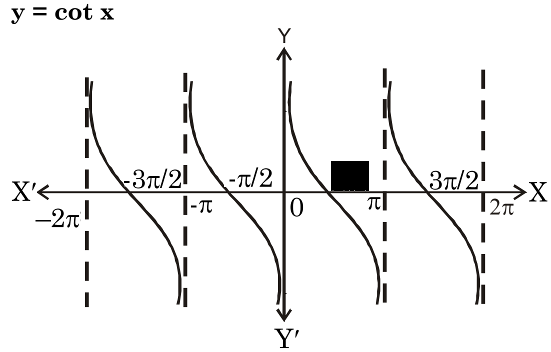
Clearly cos x is an even function therefore it is symmetrical about axis of y.

y = tan x



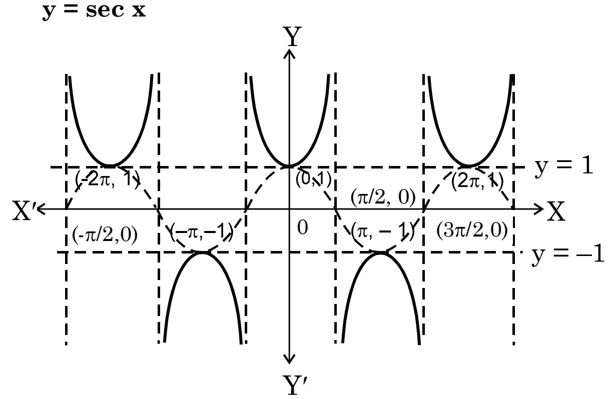
Domain : $R - \left(\frac{2n+1}{2}\right)\pi$
Range : (– ∞ , ∞)
Nature : Many one onto
Principle value : (–π/2, π/2)

y = cot x



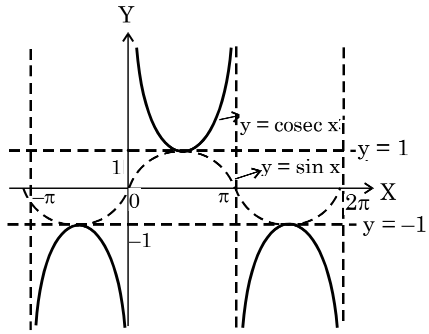
Domain : R – nπ
Range : (– ∞, ∞)
Nature : Many one onto
Principle value : (0, π)

y = sec x



Domain : R – (2n+1)π/2
Range : R – (–1, 1) or (– ∞, –1] ∪ [1, ∞)
Nature : Many one into
Principle value : same as y = cos x

y = cosec x



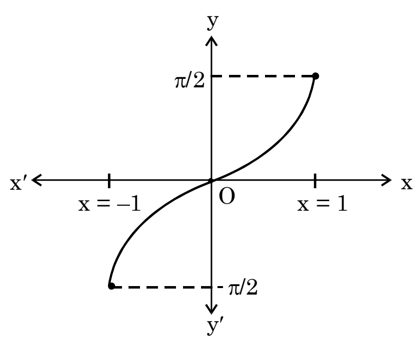
Domain	: $R - n\pi$
Range	: $R - (-1, 1)$ or $(-\infty, -1] \cup [1, \infty)$
Nature	: Many one into
Principle value	: same as y = sin x

✦ Inverse Circular Function or Inverse Trigonometric Functions

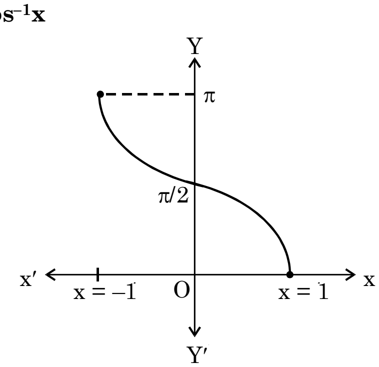
$y = \sin^{-1}x$, $y = \cos^{-1}x$, $y = \tan^{-1}x$

$y = \cot^{-1}x$, $y = \sec^{-1}x$, $y = \text{cosec}^{-1}x$

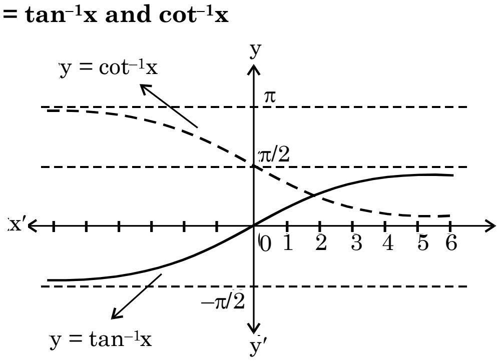
$y = \sin^{-1} x$



$y = \cos^{-1}x$

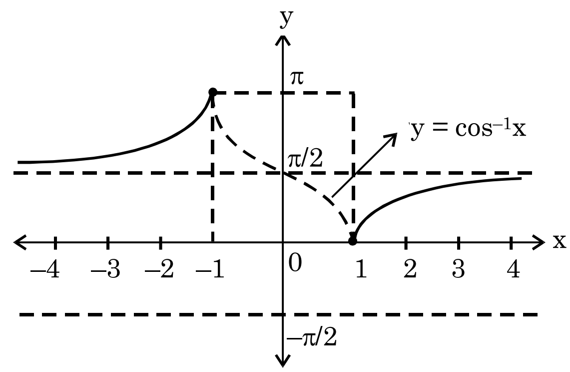


$y = \tan^{-1}x$ and $\cot^{-1}x$

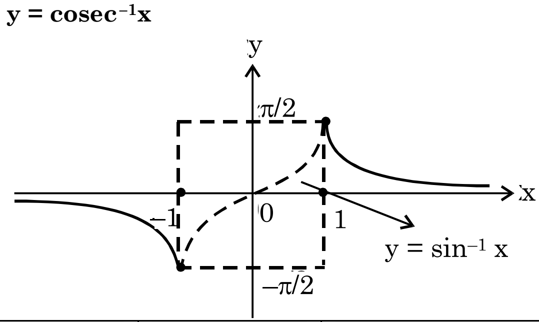


> **NOTE** **All inverse function, $f^{-1}(x)$ are drawn by taking reflection of f(x) in line y = x.**

$y = \sec^{-1}x$



$y = \text{cosec}^{-1}x$



Function	Domain	Range
$\sin^{-1} x$	[–1, 1]	[–π/2, π/2]
$\cos^{-1} x$	[–1, 1]	[0, π]
$\tan^{-1} x$	R	(–π/2, +π/2)
$\cot^{-1} x$	R	(0, π)
$\sec^{-1} x$	R – (–1, 1)	[0, π] – {π/2}
$\text{cosec}^{-1} x$	R – (–1, 1)	[–π/2, π/2] – {0}

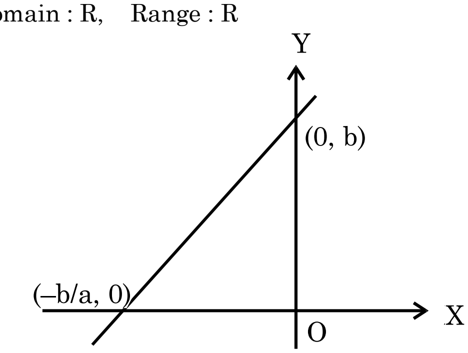
9. SOME SPECIAL FUNCTION AND THEIR GRAPHS

✦ Linear Function

$f(x) = ax + b$, $a \neq 0$ and $x \in R$

Where a and b are constant

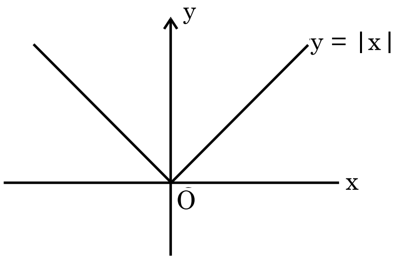
Domain : R, Range : R



✦ Modulus Function

$$f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

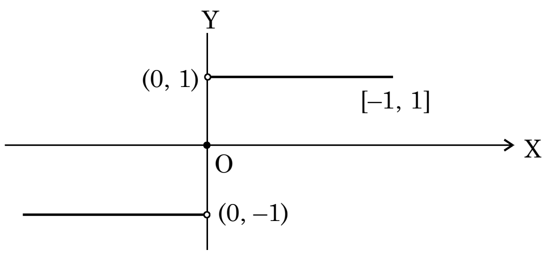
Domain : R
Range : $[0, \infty)$
It is an even, continuous and many-one function.
Graph is symmetrical with respect to y-axis.



✦ Signum Function

$$f(x) = \frac{|x|}{x}, \quad x \neq 0$$

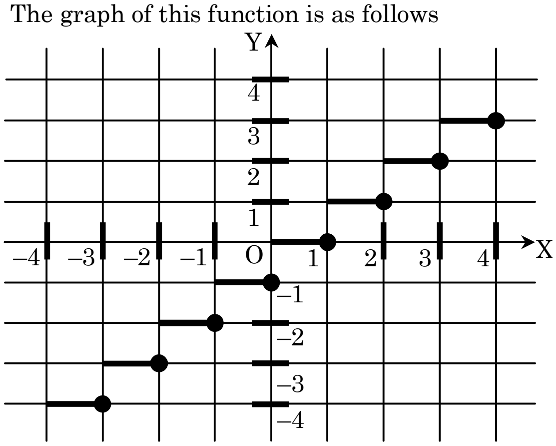
or $$f(x) = \begin{cases} -1, & x < 0 \\ 1, & x > 0 = 0 \quad x = 0 \\ 0, & x = 0 \end{cases}$$



Domain : R
Range : {–1, 0, 1}
It is a many one and discontinuous function.

✦ Greatest Integer Function

A function is said to be greatest integer function if it is of the form of f(x) = [x] where [x] = integer equal or less than x.
The graph of this function is as follows



Example. 9

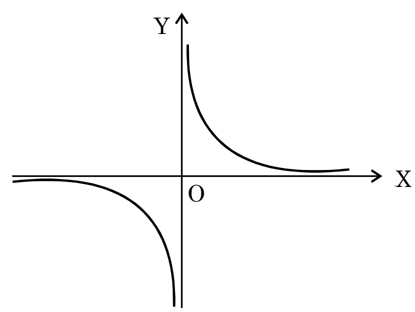
[4.2] = 4, [3.6] = 3, [– 4.4] = –5, [–5.8] = –6
f(x) = y = [x]
$0 \leq x < 1 \Rightarrow y = 0$
$1 \leq x < 2 \Rightarrow y = 1$
$2 \leq x < 3 \Rightarrow y = 2$
⋮ ⋮
and so on

NOTE Important Identities :

(i) $x - 1 < [x] \leq x$
(ii) $[x] + 1 > x$
(iii) If f (x) = [x + n], where $n \in I$ and [.] denotes the greatest integer function, then f(x) = n + [x]
(iv) x = [x] + { x } , [.] & { } denotes the integral and fractional part of x respectively.
(v) $x - 1 < [x] \leq x$
$[-x] = -[x]$, if $x \in I$
$[-x] = -[x] - 1$, if $x \notin I$
$[x] - [-x] = 2n$, if $x = n$, $n \in I$
$[x] - [-x] = 2n + 1$, if $x = n + \{x\}$, $n \in I$
$[x] \geq n \Rightarrow x \geq n$, $n \in I$
$[x] \leq n \Rightarrow x < n + 1$, $n \in I$
$[x] > n \Rightarrow x \geq n + 1$, $n \in I$
$[x] < n \Rightarrow x < n$, $n \in I$

✦ Rectangular Hyperbola

f(x) = 1/x.
Domain : R – {0}
Range : R – {0}

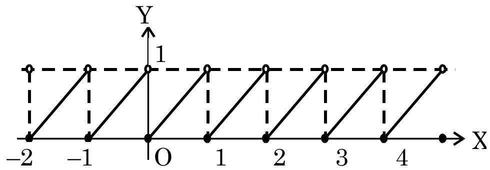


It is a continuous in its domain and one-one into function.

✦ Fractional Part of x :

$f(x) = x - [x]$, $x \in R$.
i.e. f(x) = { x }

$$f(x) = \begin{cases} x+1, & x \in [-1, 0) \\ x, & x \in [0, 1) \\ x-1, & x \in [1, 2) \\ 0, & x \in Z \end{cases}$$



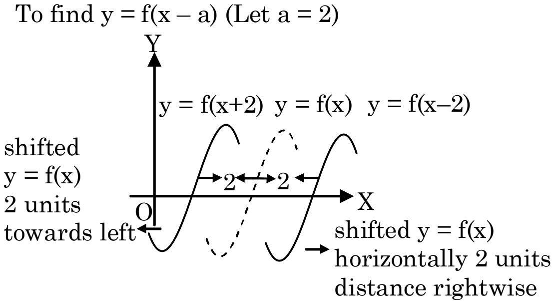
Domain : R
Range : [0, 1)
Nature : Many one into
This is a many one function with period 1. It is discontinuous at every integer.

10.TRANSFORMATION

If graph of y = f(x) be known then to find the graph of

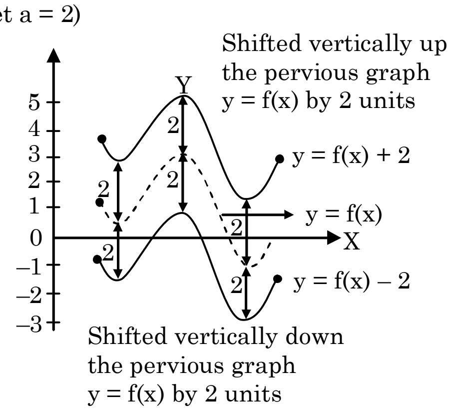
- ✦ **y = f(x – a) or y = f(x + a)**

To find y = f(x – a) (Let a = 2)

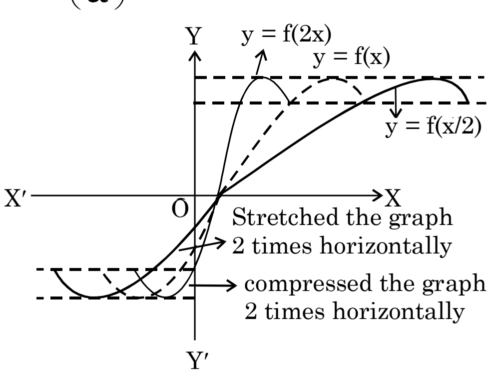


- ✦ **y = f (x) + a : or y = f (x) – a**

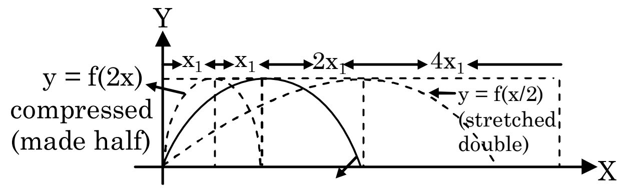
(Let a = 2)



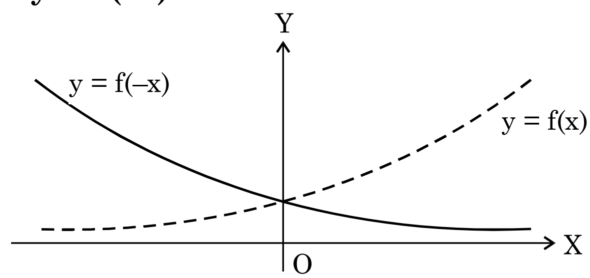
- ✦ **$y = f\left(\frac{x}{a}\right)$ or y = f(ax) : (Let a = 2, 1/2)**



See more examples about the same :



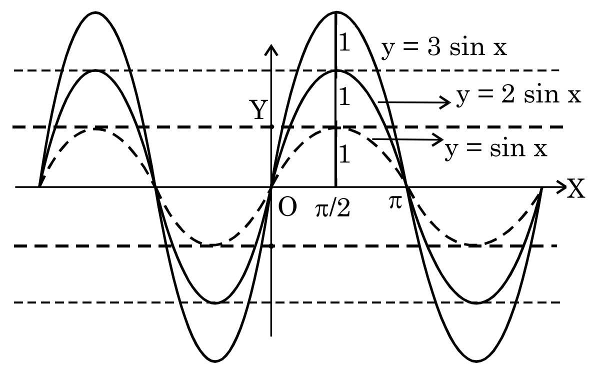
- ✦ **y = f (–x)**



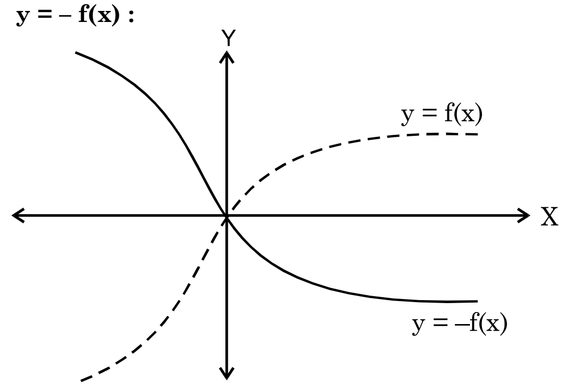
Reflection of y = f(x) w. r. t. axis of y is y = f(– x)

- ✦ **To Find y = k f (x)**

Rule – Stretch the previous graph k times vertically
e.g. see below y = 2 sin x, y = 3 sin x

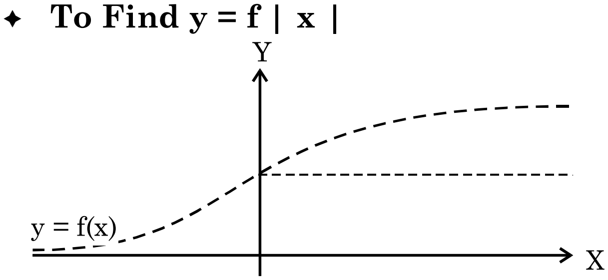


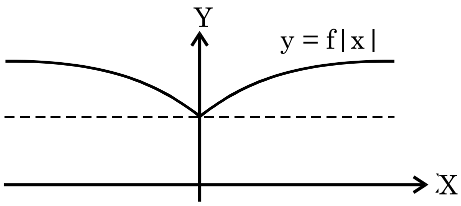
(i) y = – f(x) :



Reflection of y = f(x) w. r. t. axis of x is y = – f(x)

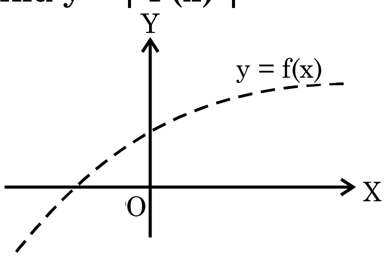
- ✦ **To Find y = f | x |**

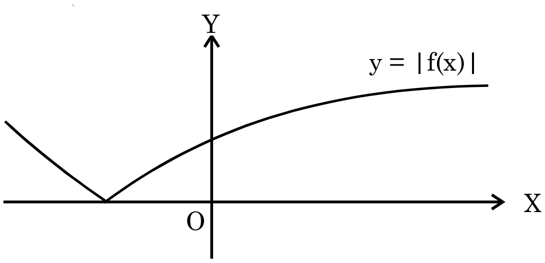




RULE :
Neglect the graph lying in II[nd] and III[rd] quadrant and,
Take the image of graph lying in I and IV[th] quadrant w. r. t. axis of y.
The original graph including its image is called y = f | x |.
Here we took the image of the portion lying in first quadrant about axis of y and left the portion which was lying in second quadrant.

✦ To Find y = | f (x) |



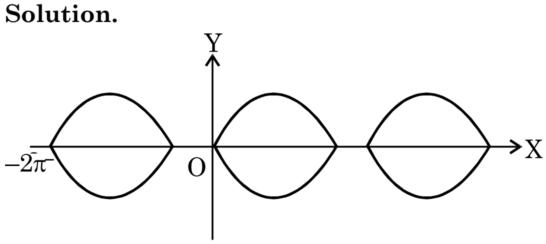


Rule : Take the image of the portion line below axis of x about axis of x. Remain as it is the portion above the axis of x.

Example Based on
Transformation

✍ Example. 10

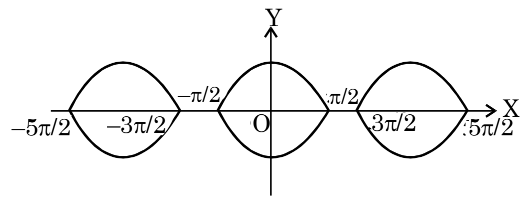
Draw the graph of | y | = sin x
Solution.



> **NOTE** ▶ **You can draw | y | = sin | x | just by taking mirror image of portion lying in I and IV quadrant w.r.t. axis of y.**

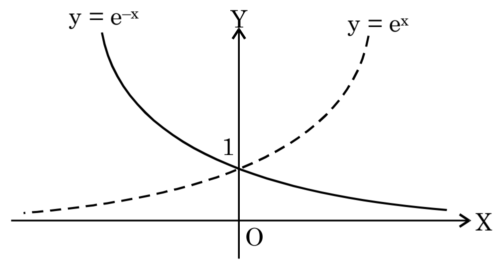
✍ Example. 11

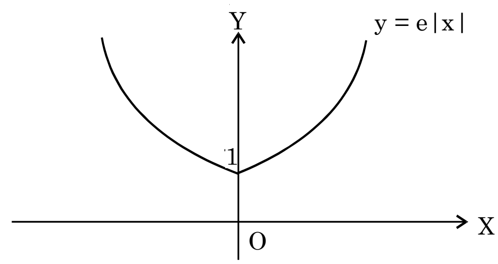
Draw the graph of | y | = cos x
Solution.

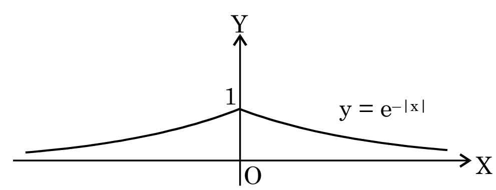


✍ Example. 12

Draw $y = e^{-x}$, $y = e^{|x|}$, $y = e^{-|x|}$.
Solution.

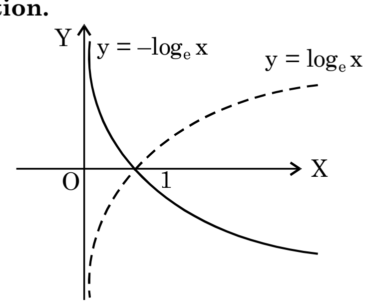


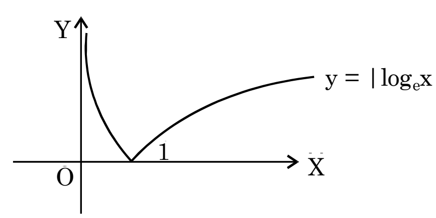


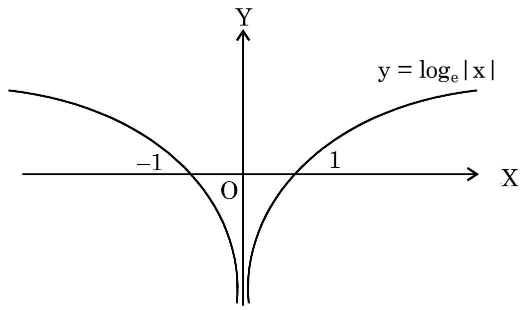


✍ Example. 13

Draw $y = -\log_e x$, $y = |\log_e x|$, $y = \log_e |x|$
Solution.







11. ELEMENTARY FUNCTION

If arithmetical operations i.e. (operation of addition, subtraction, multiplication, division) and the operation of function of a function is used finite times over basic elementary function then resulting function is called basic elementary function.

For examples :

$f(x) = \dfrac{x^2 \log_e x}{\sin x}$

$\phi(x) = \dfrac{x^2 + e^x + 5}{\sqrt{1 - \cos^{-1} x}}$

$\psi(x) = \sin(\sin \sin \log_e x)$

$\alpha(x) = \cos(\sin e^x)$

12. FURTHER REPRESENTATION OF A FUNCTION

✦ Explicit Function

If y be clearly directly defined in terms of x only i.e. $y = f(x)$. Then y is called explicit function of x.

e.g. $y = x^2$, $y = \log_e x$, $y = \tan x$

✦ Implicit Function

A function $y = f(x)$ is said to be an implicit function of x if y cannot be written in terms of x only.

e.g. $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$,
$xy = \sin(x + y)$.

✦ Parametric Function

If x and y both becomes dependent and they are defined by a new independent variable (say t) as

$x = \phi(t)$, $y = \psi(t)$, where t is parameter

such functions are called parametric function.

e. g. $x = t^2$, $y = 2t$

NOTE **These are not kinds of functions. Simply ways of representing functions.**

13. INVERSE FUNCTION

If $f : A \to B$ be a one-one onto (bijection) function, then the mapping $f^{-1} : B \to A$ which associates each element $b \in B$ with element $a \in A$, such that $f(a) = b$, is called the inverse function of the function $f : A \to B$

$f^{-1} : B \to A$, $f^{-1}(b) = a$ $f(a) = b$

When function is given in ordered pair form then its inverse is defined as -

$f^{-1} = \{(b, a) \mid (a, b) \in f\}$

NOTE

(1) For the existence of inverse function, it should be one -one and onto.

(2) One one onto functions are also called bijective. i.e. when the function is surjective as well as injective then function is said to be bijective.

✦ Properties

(a) Inverse of a bijection is also a bijection function.

(b) Inverse of a bijection is unique.

(c) $(f^{-1})^{-1} = f$

(d) If f and g are two bijections such that (gof) exists then $(gof)^{-1} = f^{-1}og^{-1}$

(e) If $f : A \to B$ is a bijection then $f^{-1} : B \to A$ is an inverse function of f .
$f^{-1}of = I$ and $fof^{-1} = I$.
Here I, is an identity function (y = x is called an Identity function) i.e.,
$f^{-1}of = fof^{-1} = x$.

Example Based on

Inverse Function

Example. 14

If $f : R \to R$, $f(x) = 2x + 3$, then find $f^{-1}(x)$

Solution.

Since f is a bijection therefore its inverse mapping exists if

$y = 2x + 3 \Rightarrow x = (y - 3)/2$

$\Rightarrow y = \dfrac{x-3}{2}$ $\therefore f^{-1}(x) = \dfrac{x-3}{2}$

Example. 15

If $f : R \to R$ where $f(x) = x^2 + 3x + 7$ then find $f^{-1}(7)$.

Solution.

Since $y = f(x)$ is many one therefore inverse of f(x) will not exist i.e. $f^{-1}(7) = \phi$.

Example. 16

If $f : R \to R$, $f(x) = x^3 + 2$ then find $f^{-1}(x)$.

Solution.

$f(x) = x^3 + 2$, $x \in R$.

Since this is a one–one onto function therefore inverse of this function (f^{-1}) exists.

Let $f^{-1}(x) = y$

$x = f(y) \Rightarrow x = y^3 + 2 \Rightarrow y = (x - 2)^{1/3}$

$f^{-1}(x) = (x - 2)^{1/3}$.

14. COMPOSITE FUNCTION

Let A, B and C be three non void sets and let $f : A \to B$, $g : B \to C$ be two functions. Since f is a function from A to B, therefore for each $x \in A$ there exists a unique element $g(f(x)) \in C$.

Thus, for each $x \in A$ there exists a unique element $g(f(x)) \in C$.

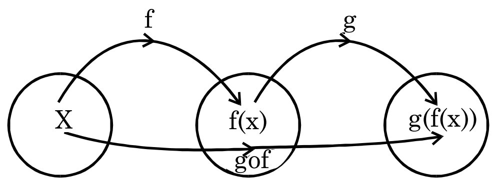
If follows from the above discussion that f and g when considered together define a new function from A to C. This function is called the compositions of f and g and is denoted by gof . We define it formally as follows -

Definition :

Let $f : A \to B$ and $g : B \to C$ be two functions.

Then a function $gof : A \to C$ defined by

$gof(x) = g(f(x))$ for all $x \in A$ is called the composition of f and g.



NOTE **It is evident from the definition that fog is defined only if for each x ∈ A, f(x) is an element of g so that we can take its g - image. Hence, for the composition gof to exist, the range of f must be a subset of the domain of g.**

15. PERIODIC FUNCTION

A function on y = f(x) is said to be periodic. If there exist a series of integral multiple of positive constant (say T) such that

$f(x + T) = f(x + 2T) = f(x + 3T) = ... f(x + nT) = f(x)$.

Then period of f(x) is T, 2T, 3T, 4T, ... nT. But fundamental period is T. In numerical problems if the word '**Period**' comes sense goes to fundamental period.

Points to Remember

The function whose period is 2π

1. $(\sin x)^{2n+1}, (\cos x)^{2n+1}, (\sec x)^{2n+1}, (\text{cosec } x)^{2n+1}$

The function whose period is π

2. $(\sin x)^{2n}, (\cos x)^{2n}, (\sec x)^{2n}, (\text{cosec } x)^{2n}$
3. $(\tan x)^n, (\cot x)^n$
4. $|\sin x|, |\cos x|, |\tan x|, |\cot x|, |\sec x|, |\text{cosec } x|$
5. If f(x) has the period T, then $f(\pm ax + b)$ will have the period $\frac{T}{|a|}$
6. If $f_1(x)$ has the period T_1

   $f_2(x)$ has the period T_2

   Then period of $af_1(x) + bf_2(x)$ will be LCM of T_1 and T_2, provided there should not exist a number 'r' such that

   $f_1(x + r) = f_2(x)$ & $f_2(x + r) = f_1(x)$ and $r \not< \text{LCM}$ of T_1 and T_2.

   (In this case period = r) (**Note it**)

   e.g. The period of $|\sin x| + |\cos x|$ is $\pi/2$.

   LCM of two or more fractional number.

   $$\text{LCM of } \frac{a}{b}, \frac{c}{d}, \frac{e}{f} = \frac{\text{LCM of }(a,c,e)}{\text{HCF}(b,d,f)}$$
7. Period of f (x) = T

   $\Rightarrow$ Period of 1/f(x) = T

Example Based on

Periodic Function

Example. 17

Find the period of $f(x) = x + \sin x - [x]$

Solution.

Given $f(x) = \sin x + \{x\}$

$= g(x) + h(x)$

Period of $g(x) = 2\pi$

Period of $h(x) = 1$

2π is irrational and 1 is rational. Therefore LCM will not exist. Note that if irrational quantities be like π, 2π or $\pi, \frac{\pi}{3}, \frac{\pi}{6}$ or $\sqrt{3}$, $3\sqrt{3}$, $9\sqrt{3}$. LCM or HCF is existing i.e. if there be multiple of a particular irrational quantities then LCM or HCF exist.

Example. 18

Find the period of $f(x) = x - [x - m]$, $m \in I$., where [ ] denotes the greatest integer function.

Solution.

Given $f(x) = x - [x - m]$

$= (x - m) - [x - m] + m$

$= \{x - m\} + m$

$\because \{x - m\}$ is periodic

$\therefore m + \{x - m\}$ will also be periodic and period will be 1.

$\because$ Period of $\{ x \}$ = Period of $\{ x - a\} = 1$

Example. 19

Draw the graph of $y = [\sin x]$ and also find the period, if possible.

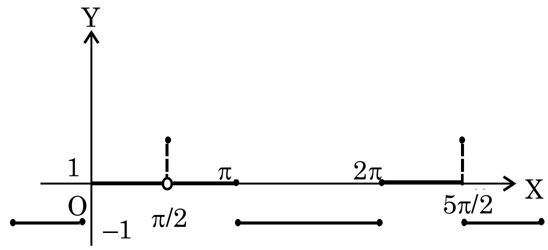
Solution.

$[\sin x] = 0, \; 0 \le x < \frac{\pi}{2}$

$= 1, \; x = \frac{\pi}{2}$

$= 0, \; \frac{\pi}{2} < x < \pi$

$= -1, \; \pi \le x \le \frac{3\pi}{2}$, ...



Clearly f(x) is periodic and its period is 2π.

Example. 20

Draw $f(x) = \sin\left[\frac{x}{\pi}\right]$ Is it periodic ? If periodic find its period.

Solution.

$0 \le x < \pi \Rightarrow \left[\frac{x}{\pi}\right] = 0$

$\pi \le x < 2\pi \Rightarrow \left[\frac{x}{\pi}\right] = 1$

$2\pi \le x < 3\pi \Rightarrow \left[\frac{x}{\pi}\right] = 2$

$3\pi \le x < 4\pi \Rightarrow \left[\frac{x}{\pi}\right] = 3$

Clearly f(x) is not periodic.

16.EVEN AND ODD FUNCTIONS

✦ Definition

If, f(–x) = f(x), then y = f(x) is said to be even function and if f (–x) = – f (x), then y = f(x) is called an odd function.

✦ Properties of Even and Odd Function

(a) The product of two even functions is even function.

(b) The sum and difference of two even functions is even function.

(c) The sum and difference of two odd functions is odd function.

(d) The product of two odd functions is even function.

(e) The product of an even and an odd function is odd function.

(f) The sum of even and odd function is neither even nor odd function.

(g) It is not essential that every function is even or odd. It is possible to have some functions which are neither even nor odd function.

(h) Even functions are symmetric w.r.t. y-axis and odd functions are symmetric w.r.t. origin.

e.g. $f(x) = x^2 + x^3$, $f(x) = \log_e x$, $f(x) = e^x$

Neither odd nor even functions.

e.g. $f(x) = \cos x$, $f(x) = \sec x$, $f(x) = x^4$, ... these are the examples of even functions.

e.g. $f(x) = \sin x$, $f(x) = \cot x$, $f(x) = x^3$, ... these are the examples of odd functions.

NOTE **Zero function f(x) = 0** is the only function which is even and odd both.

Example Based on

Even and Odd Function

Example. 21

If $f(x) = \frac{a^x + 1}{a^x - 1}$ then is it an even or odd function -

Solution.

$$f(-x) = \frac{a^{-x} + 1}{a^{-x} - 1} = -\frac{a^x + 1}{a^x - 1}$$

∴ $f(-x) = -f(x)$

∴ f(x) is an odd function.

17.ALGEBRAIC FUNCTION

✦ Rational Integral Function or Polynomial:

A function having the form

$y = a_0 + a_1x + a_2x^2 + a_3x^3 + ... a_nx^n$, where a_0, a_1, a_2 .... a_n

are real constant called rational integral function or polynomial of degree n.

✦ Fractional Rational Function :

The ratio of two polynomial is called Fraction Rational function or simply rational function.

e.g. $y = \frac{x^{12} + x^2 - 1}{x^6 + x^4 + 1}$

✦ Irrational Function :

Functions with operations of addition, subtraction, multiplication, division and raising to power with non-integral rational exponent are called irrational functions.

$y = \sqrt{x}$, $y = \frac{\sqrt{x^3 + 1} - \sqrt{x^{11}}}{\sqrt{x^2 + x + 1}}$

$$y = \frac{x^{17/3} + x^{103/7} - x}{\sqrt[3]{x^{17} + x^5 - 3}}$$

$$y = \frac{\sqrt{\sqrt{x^2 + 5} + x^{16.5} + x^{1/3}}}{\sqrt{x^2 - 7} - \sqrt{\sqrt{x^{2/3} - 1}}}$$

Such type of function are called **Irrational function**.

- **Transcedental Function :**
  The function which are not algebraic called transcedental function.

  e.g. $f(x) = \sin x$

  $y = \cos^{-1}x$

  $y = \log_e x,\ y = \sqrt{\log_e x - \sin^{-1} x}$

  $y = \dfrac{\log_e x + \tan x}{\sin^{-1} x + 2^x}$ etc.

18.BOUNDED FUNCTION

The function f(x) is said to be bounded above if there exists M such that $y = f(x) \not> M$ (i.e. not greater than M) for all x of the domain and M is called upper bound. Similarly f(x) is said to be bounded below if there exists m such that $y = f(x) \not< m$ (i.e. never less than m) for all x of the domain and m is called the lower bound.
If however, there does not exist M and m as stated above, the function is said to be unbounded.

19.EXTENSION OF A FUNCTION

Let $f : A \to B$ s. t. $f(x) = y\ \forall\ x \in A$
If $X \supset A$ i.e. X is a super set of A and $Y \supset f(A)$ then another function

$g : X \to Y$ s.t. $g(x) = f(x)\ \forall\ x \in A$

is called an extension of f from A to X.

- **Even and Odd extension**
  Let f(x) be a function defined on A = [0, a] and X = [–a, a] is a super set of A then an extension of f(x) on X = [–a, a] will be even or odd extension if f(x) becomes an even or odd function on X.

Example Based on

Extension of Function

Example. 22

Let $f(x) = x^2 + 5x - 2$ defined on A = [0, 2]. Find even and odd extension of f(x) in [–2, 2]

Solution.

$f(x) = x^2 + 5x - 2,\quad f(-x) = x^2 - 5x - 2$

Let g_e and g_o denote even and odd extension

$$g_e(x) = \begin{cases} f(x) : x \in [0, 2] \\ f(-x) : x \in [-2, 0)\ [\because f(-x) = f(x) \text{ for even}] \end{cases}$$

$$g_o(x) = \begin{cases} f(x) : x \in [0, 2] \\ -f(-x) : x \in [-2, 0)\ [\because -f(-x) = f(x) \text{ for odd}] \end{cases}$$

$$g_e(x) = \begin{cases} x^2 + 5x - 2 & : x \in [0, 2] \\ x^2 - 5x - 2 & : x \in [-2, 0[\end{cases}$$

$$g_o(x) = \begin{cases} x^2 + 5x - 2 & : x \in [0, 2] \\ -x^2 + 5x + 2 & : x \in [-2, 0[\end{cases}$$

SOLVED EXAMPLES

Ex.1 Find the domain and range of the function $f(x) = \sqrt{2-x} + \sqrt{1+x}$

Sol. Domain of $f(x) = \{x \mid 2 - x \ge 0 \text{ and } 1 + x \ge 0\}$

$\therefore$ domain of $f(x) = [-1, 2]$

Again, $\{f(x)\}^2 = \left(\sqrt{2-x} + \sqrt{1+x}\right)^2$

$= 3 + 2\sqrt{(2-x)(1+x)}$

$= 3 + 2\sqrt{2+x-x^2}$

$= 3 + 2\sqrt{\frac{9}{4} - \left(x - \frac{1}{2}\right)^2}$

$\therefore$ the greatest value of $\{f(x)\}^2$

$= 3 + 2.\sqrt{\frac{9}{4}} = 6$, when $x = \frac{1}{2}$

the least value of $\{f(x)\}^2 = 3 + 0 = 3$,

when $x - \frac{1}{2} = \frac{3}{2}$, i.e. $x = 2$

$\therefore$ the greatest value of $f(x) = \sqrt{6}$

and the least value of $f(x) = \sqrt{3}$

$\therefore$ range of $f(x) = [\sqrt{3}, \sqrt{6}]$

Ex.2 Find the range of the following function

$$f(x) = \frac{3}{2-x^2}$$

Sol. Let $y = \frac{3}{2-x^2} = f(x)$ ....(1)

The function y is not defined for

$x = \pm\sqrt{2}$

From (1), $x^2 = \frac{2y-3}{y}$

since for real x, $x^2 \ge 0$,

We have $\frac{2y-3}{y} \ge 0$

$\therefore$ $y \ge 3/2$ or $y < 0$ (Note that $y \ne 0$)

Hence the range of the function is

$]-\infty, 0[\cup [3/2, \infty)$

Ex.3 Find the range of the following function:

$$f(x) = \log_2\left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}}\right)$$

Sol. $\because$ $f(x) = \log_2\left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}}\right)$

$= \log_2\left(\sin\left(\pi - \frac{\pi}{4}\right) + 3\right) = y$ (let)

$\Rightarrow$ $2^y = \sin\left(\pi - \frac{\pi}{4}\right) + 3$

$\Rightarrow$ $2^y - 3 = \sin\left(\pi - \frac{\pi}{4}\right)$

But $-1 \le \sin\left(\pi - \frac{\pi}{4}\right) \le 1$

$\therefore$ $-1 \le 2^y - 3 \le 1$

$\Rightarrow$ $2 \le 2^y \le 4$

$\Rightarrow$ $2^1 \le 2^y \le 2^2$

Hence $y \in [1, 2]$.

Hence Range of $f(x)$ is $[1, 2]$.

Ex.4 Find the period of the following function

$f(x) = e^{x - [x] + |\cos \pi x| + |\cos 2\pi x| + ...+ |\cos n\pi x|}$,

[.] is greatest integer function.

Sol. $f(x) = e^{x - [x] + |\cos \pi x| + |\cos 2\pi x| + ...+ |\cos n\pi x|}$

Period of $x - [x] = 1$

Period of $|\cos \pi x| = 1$

Period of $|\cos 2\pi x| = 1/2$

....

....

Period of $|\cos n\pi x| = 1/n$

So period of $f(x)$ will be

L.C.M. of all periods = 1.

Ex.5 Let a function $f : R \to R$ be defined as $f(x) = x - [x]$, (where $[x]$ is a greatest integer $\le x$), for all $x \in$ R, Is the function bijective ?

Sol. Let $x_1, x_2 \in$ R

$f(x_1) = f(x_2) \Rightarrow x_1 - [x_1] = x_2 - [x_2]$

$\Rightarrow x_1 \ne x_2$

$\therefore$ The function is not bijective.

Ex.6 If $f(x) = \begin{cases} x^3 + 1, & x < 0 \\ x^2 + 1, & x \ge 0 \end{cases}$,

$g(x) = \begin{cases} (x-1)^{1/3}, & x < 1 \\ (x-1)^{1/2}, & x \ge 1 \end{cases}$

Compute $gof(x)$.

Sol. We have

$gof(x) = g(f(x))$

Ex.7 The value of $n \in$ I for which the function $f(x) = \frac{\sin nx}{\sin(x/n)}$ has 4π as its period is -

(A) 2 (B) 3 (C) 5 (D) 4

Sol. For $n = 2$,

we have $\dfrac{\sin 2x}{\sin(x/2)} = 4(\cos x/2)\cos x$.

The period of cosx is 2π, & that of $\cos(x/2)$ is 4π.

Hence the period of $\dfrac{\sin 2x}{\sin(x/2)}$ is 4π.

Also, the period of $\dfrac{\sin 3x}{\sin(x/3)}, \dfrac{\sin 5x}{\sin(x/5)}$ and $\dfrac{\sin 4x}{\sin(x/4)}$ cannot be 4π. **Ans.[A]**

Ex.8 Prove that even functions do not have inverse.

Sol. Even functions are many one function and for the existence of inverse function should be one-one. Hence inverse of an even function will not exist.

Ex.9 Prove that periodic functions do not have inverse.

Sol. $f(x)$ is periodic

$\Rightarrow$ f is many one

$\Rightarrow$ f^{-1} does not exist.

$$= \begin{cases} g(x^3+1), & x<0 \\ g(x^2+1), & x \ge 0 \end{cases}$$

$$= \begin{cases} (x^3+1-1)^{1/3} & x<0 \\ (x^2+1-1)^{1/2}, & x \ge 0 \end{cases}$$

$$= \begin{cases} x, & x<0 \\ x, & x \ge 0 \end{cases} = x \text{ for all } x.$$

Hence, $gof(x) = x$ for all x.

Ex.10 Show that the function $f : R \to R$ defined by $f(x) = 3x^3 + 5$ for all $x \in R$ is a bijection.

Sol. Injectivity : Let x, y be any two elements of R(domain).

Then,

$f(x) = f(y) \Rightarrow 3x^3 + 5 = 3y^3 + 5$

$\Rightarrow x^3 = y^3 \Rightarrow x = y$

Thus, $f(x) = f(y)$

$\Rightarrow x = y$ for all $x, y \in R$.

so, f is an injective map.

Surjectivity : Let y be an arbitrary element of R(co-domain).

Then,

$f(x) = y \Rightarrow 3x^3 + 5 = y \Rightarrow x^3 = \dfrac{y-5}{3}$

$\Rightarrow x = \left(\dfrac{y-5}{3}\right)^{1/3}$

Thus we find that for all $y \in R$ (co-domain) there exists $x = \left(\dfrac{y-5}{3}\right)^{1/3} \in R$ (domain) such that

$$f(x) = f\left(\left(\frac{y-5}{3}\right)^{1/3}\right) = 3\left[\left(\frac{y-5}{3}\right)^{1/3}\right]^3 + 5$$

$= y - 5 + 5 = y$

This shows that every element in the co-domain has its pre-image in the domain. So, f is a surjection. Hence, f is a bijection.

Ex.11 Let,
$f(x) = x + 1, \quad x \le 1$
$\quad = 2x + 1, \quad 1 < x \le 2$
$g(x) = x^2, \quad -1 \le x < 2$
$\quad = x + 2, \quad 2 \le x \le 3$

Find fog and gof.

Sol.
$f\{g(x)\} = g(x) + 1, \quad g(x) \le 1$
$\quad = 2g(x) + 1, \quad 1 < g(x) \le 2$
$\Rightarrow f\{g(x)\} = x^2 + 1, \quad -1 \le x \le 1$
$\quad = 2x^2 + 1, \quad 1 < x \le \sqrt{2}$
$g\{f(x)\} = \{f(x)\}^2, \quad -1 \le f(x) < 2$
$\quad = f(x) + 2, \quad 2 \le f(x) \le 3$
$gof(x) = (x+1)^2, \quad -2 \le x < 1$
$\quad = (x+1)^2 \quad -2 \le x \le 1$

Ex.12 Find the inverse of the following function :

$$f(x) = \begin{cases} x, & x<1 \\ x^2, & 1 \le x \le 4 \\ 8\sqrt{x}, & x>4 \end{cases}$$

Sol. Let $f(x) = \begin{cases} x, & x<1 \\ x^2, & 1 \le x \le 4 \\ 8\sqrt{x}, & x>4 \end{cases}$

Let $f(x) = y \quad \therefore x = f^{-1}(y)$

$$\Rightarrow x = \begin{cases} y, & y<1 \\ \sqrt{y}, & 1 \le y \le 16 \\ y^2/64, & y>16 \end{cases}$$

$$\Rightarrow f^{-1}(y) = \begin{cases} y, & y<1 \\ \sqrt{y}, & 1 \le y \le 16 \\ y^2/64, & y>16 \end{cases}$$

$$\Rightarrow f^{-1}(x) = \begin{cases} x, & x<1 \\ \sqrt{x}, & 1 \le x \le 16 \\ x^2/64, & x>16 \end{cases}$$

Ex.13 Let $f(x) = x^2 + x$ be defined on the interval [0, 2]. Find the odd and even extensions of $f(x)$ in the interval [–2, 2].

Sol. Odd extension.

$$f(x) = \begin{cases} f(x), & 0 \le x \le 2 \\ -f(-x), & -2 \le x < 0 \end{cases}$$

$$= \begin{cases} x^2 + x, & 0 \le x \le 2 \\ -x^2 + x, & -2 \le x < 0 \end{cases}$$

Even extension

$$f(x) = \begin{cases} f(x), & 0 \le x \le 2 \\ f(-x), & -2 \le x < 0 \end{cases}$$

$$= \begin{cases} x^2 + x, & 0 \le x \le 2 \\ x^2 - x, & -2 \le x < 0 \end{cases}$$

Ex.14 Let $f : R \to R$ be given by $f(x) = (x + 1)^2 - 1$, $x \ge -1$. Show that f is invertible. Also, find the set $S = \{x : f(x) = f^{-1}(x)\}$.

Sol. In order to show that $f(x)$ is invertible, it is sufficient to show that $f(x)$ is a bijection.

f is an injection : For any $x, y \in R$ satisfying $x \ge -1, y \ge -1$,

We have $f(x) = f(y)$

$\Rightarrow (x + 1)^2 - 1 = (y + 1)^2 - 1$

$\Rightarrow x^2 + 2x = y^2 + 2y$

$\Rightarrow x^2 - y^2 = -2(x - y)$

$\Rightarrow (x - y)(x + y) = -2(x - y)$

$\Rightarrow (x - y)[x + y + 2] = 0$

$\Rightarrow x - y = 0$ or $x + y + 2 = 0$

$\Rightarrow x = y$ or $x = y = -1$

Thus, $f(x) = f(y) \Rightarrow x = y$ for all $x \ge -1, y \ge -1$.

So, $f(x)$ is an injection.

f is a surjection : For all $y \ge -1$ there exists.

$x = -1 + \sqrt{y+1} \ge -1$ such that $f(x) = y$

So, $f(x)$ is a surjection.

Hence, f is a bijection. Consequently, it is invertible.

$f(x) = f^{-1}(x) \quad \Rightarrow f(x) = x$

$(x + 1)^2 - 1 = x \quad \Rightarrow x = 0, -1$

Ex.15 If f, g, h are function from R to R such that

$f(x) = x^2 - 1$, $g(x) = \sqrt{x^2+1}$,

$h(x) = 0$, if $x \le 0$

$\quad = x$, if $x \ge 0$

then find the composite function $ho(fog)$ and determine whether the function fog is invertible and the function h is the identity function.

Sol. Here $f(x) = x^2 - 1$ for all x

and $g(x) = \sqrt{x^2+1}$ for all x

$\therefore \quad f\{g(x)\} = \{g(x)\}^2 - 1$

$\qquad = x^2 + 1 - 1 = x^2$ for all x

$\therefore \quad h\{f(g(x))\} = h(x^2) = x^2$

because $x^2 \ge 0$ [from definition of $h(x)$.]

Now, $\quad f\{g(x)\} = x^2$ for all x

As $\quad x^2 \ge 0$, $(fog)(x)$ cannot be negative.

So $\quad fog$ is not an onto function.

Hence fog is not invertible.

Again, $h(x) = x$ for $x \ge 0$.

But, by definition $h(x) \ne x$ for $x < 0$.

Hence h is not the identity function.

Ex.16 Let $f(x) = \tan x$, $x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$ and $g(x) = \sqrt{1-x^2}$. Determine fog and gof.

Sol. From the given domain of f $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$ we conclude that its range $]-\infty, \infty[$ i.e. whole of R

Domain of g is $1 - x^2 \ge 0$ or $x^2 - 1 \le 0$

or $(x + 1)(x - 1) \le 0$ or $-1 \le x \le 1$or $[-1, 1]$

and for range of g, $y = \sqrt{1-x^2}$

since $x^2 \le 1 \quad \therefore \quad y \in [0, 1]$

$(fog)\,x \quad = f(g(x)) = f\{\sqrt{1-x^2}\}$

$\qquad = f(t)$, where $t = \sqrt{1-x^2} \in [0, 1]$

range of $g \subset \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$ which is domain of f.

$\qquad = \tan t = \tan \sqrt{1-x^2}$

$(gof)\,x = g(f(x)) = g(\tan x) = g(t)$ where $t = \tan x \in$ range of $f = R$. But R is not a subset of domain of $g = [-1, 1]$

Hence gof is not defined.

Ex.17 Let $f_1(x) = \frac{x}{3} + 10$ for all $x \in R$, and $f_n(x) = f_1(f_{n-1}(x))$ for $n \ge 2$. Then find $f_n(x)$.

Sol. We have

$f_n(x) = f_1(f_{n-1}(x)), \ n \ge 2$

$\Rightarrow f_2(x) = f_1(f_1(x)) = \frac{1}{3} f_1(x) + 10$

$$= \frac{1}{3}\left(\frac{x}{3} + 10\right) + 10$$

$$= \frac{x}{3^2} + \frac{10}{3} + 10$$

$f_3(x) = f_1(f_2(x))$

$$= \frac{1}{3} f_2(x) + 10$$

$$= \frac{1}{3}\left(\frac{x}{3^2} + \frac{10}{3} + 10\right) + 10$$

$$= \frac{x}{3^3} + \frac{10}{3^2} + \frac{10}{3} + 10$$

$$f_4(x) = f_1(f_3(x)) = \frac{1}{3} f_3(x) + 10$$

$$= \frac{1}{3}\left(\frac{x}{3^3} + \frac{10}{3^2} + \frac{10}{3} + 10\right) + 10$$

$$= \frac{x}{3^4} + \frac{10}{3^3} + \frac{10}{3^2} + \frac{10}{3} + 10$$

Continuing in this manner, we obtain

$$f_n(x) = \frac{x}{3^n} + \frac{10}{3^{n-1}} + \frac{10}{3^{n-2}} + ... + \frac{10}{3} + 10$$

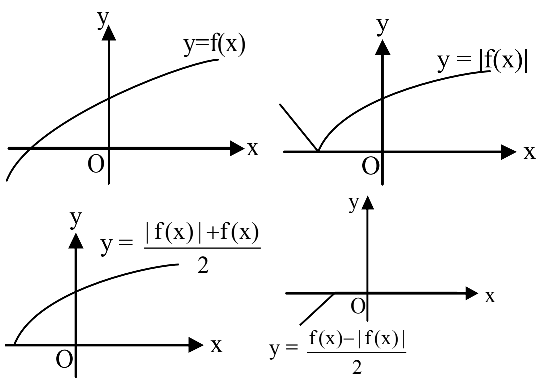
$$= \frac{x}{3^n} + 10\left(\frac{1-\frac{1}{3^n}}{1-\frac{1}{3}}\right)$$

$$= \frac{x}{3^n} + 15\left(1 - \frac{1}{3^n}\right) = \frac{x-15}{3^n} + 15$$

Ex.18 Knowing the graph of $y = f(x)$ draw

$y = \dfrac{f(x)+|f(x)|}{2}$ and $y = \dfrac{f(x)-|f(x)|}{2}$

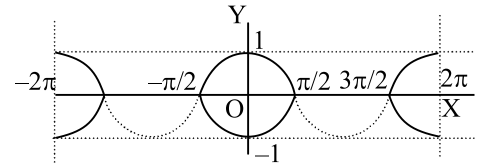
Sol. Let graph



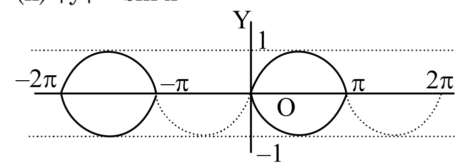
Ex.19 Draw following graphs:

(i) $|y| = \cos x$ (ii) $|y| = \sin x$

Sol. (i) $|y| = \cos x$



(ii) $|y| = \sin x$



EXERCISE (Level-1)

Question based on Domain

Q.1 Domain of $y = \log_{10}\left(\frac{5x - x^2}{4}\right)$ is
(A) (0, 5)
(B) [1, 4]
(C) $(-\infty, 0) \cup (5, \infty)$
(D) $(-\infty, 1) \cup (4, \infty)$

Q.2 The domain of definition of
$f(x) = \frac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{x^2 + 2x + 8}}$ is:
(A) (1, 4) (B) (–2, 4)
(C) (2, 4) (D) $[2, \infty)$

Q.3 The function
$f(x) = \cot^{-1}\sqrt{(x+3)x} + \cos^{-1}\sqrt{x^2 + 3x + 1}$ is
defined on the set S, where S is equal to:
(A) {0, 3} (B) (0, 3)
(C) {–3, 0} (D) [–3, 0]

Q.4 The domain of $\sqrt{\sec^{-1}\left(\frac{2 - |x|}{4}\right)}$ is
(A) R (B) R – (–1, 1)
(C) R – (–3, 3) (D) R – (– 6, 6)

Q.5 The domain of the function
$f(x) = {}^{24-x}C_{3x-1} + {}^{40-6x}C_{8x-10}$ is -
(A) {2, 3} (B) {1, 2, 3}
(C) {1, 2, 3, 4} (D) None of these

Q.6 Domain of the function
$f(x) = (1 - 3x)^{1/3} + 3\cos^{-1}\left(\frac{2x-1}{3}\right) + 3^{3\tan^{-1}x}$ is
(A) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ (B) $\left[-\frac{1}{2}, 1\right]$
(C) [–1, 2] (D) $\left[-\frac{1}{4}, \frac{1}{2}\right]$

Q.7 The function $f(x) = \frac{\sec^{-1}x}{\sqrt{x - [x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to -
(A) R
(B) R – $\{(-1, 1) \cup \{n : n \in Z\}\}$
(C) $R^+ - (0, 1)$
(D) $R^+ - \{n : n \in N\}$

Q.8 The function
$f(x) = \cos^{-1}\left(\frac{|x| - 3}{2}\right) + [\log_e(4 - x)]^{-1}$ is
defined for -
(A) $[-1, 0] \cup [1, 5]$
(B) $[-5, -1] \cup [1, 4]$
(C) $[-5, -1] \cup ([1, 4) - \{3\})$
(D) [1, 4] – {3}

Q.9 The domain of function $f(x) = \log |\log x|$ is-
(A) $(0, \infty)$ (B) $(1, \infty)$
(C) $(0, 1) \cup (1, \infty)$ (D) $(-\infty, 1)$

Q.10 The domain of function
$f(x) = \frac{1}{\log_{10}(3 - x)} + \sqrt{x + 2}$ is -
(A) [–2, 3) (B) [–2, 3) – {2}
(C) [–3, 2] (D) [–2, 3] – {2}

Question based on Range

Q.11 The range of the function $y = \frac{1}{2 - \sin 3x}$ is :
(A) $\left(\frac{1}{3}, 1\right)$ (B) $\left[\frac{1}{3}, 1\right)$
(C) $\left[\frac{1}{3}, 1\right]$ (D) None of these

Q.12 The value of the function
$f(x) = \frac{x^2 - 5x + 6}{x^2 - 4x + 3}$ lies in the interval -
(A) $(-\infty, \infty) - \left\{\frac{1}{2}, 1\right\}$ (B) $(-\infty, \infty)$
(C) $(-\infty, \infty) - \{1\}$ (D) None of these

Q.13 The range of the function,
$y = \log_{\sqrt{7}}(\sqrt{2}(\sin x - \cos x) + 5)$ is
(A) R (B) Z
(C) $[\log_7 4, \log_7 5]$ (D) $[2 \log_7 3, 2]$

Q.14 Which of the following function(s) has the range [–1, 1]
(A) $f(x) = \cos(2 \sin x)$
(B) $g(x) = \cos\left(1 - \frac{1}{1 + x^2}\right)$
(C) $h(x) = \sin(\log_2 x)$
(D) $k(x) = \tan(e^x)$

Q.15 The range of the function $f(x) = \cos(\cos^{-1}\{x\})$ is (where $\{\cdot\}$ denotes the fractional part function)
(A) [0, 1)   (B) [0, 1]
(C) (0, 1)   (D) (0, 1]

Q.16 Let $f(x) = \dfrac{x-[x]}{1-[x]+x}$, then range of $f(x)$ is-
(where [ ] represent greatest integer function)
(A) [0, 1]   (B) $\left[0, \dfrac{1}{2}\right]$
(C) $\left[\dfrac{1}{2}, 1\right]$ (D) $\left[0, \dfrac{1}{2}\right)$

Q.17 The range of the function $y = \log_3(5 + 4x - x^2)$ is-
(A) (0, 2]   (B) $(-\infty, 2]$
(C) (0, 9]   (D) None of these

Question based on Classification of functions

Q.18 Let $f : R \to R$ be a function defined by
$f(x) = \dfrac{x^2+2x+5}{x^2+x+1}$ is :
(A) one-one and into
(B) one-one and onto
(C) many-one and onto
(D) many-one and into

Q.19 The function $f : [2, \infty) \to$ Y defined by $f(x) = x^2 - 4x + 5$ is both one–one & onto if:
(A) Y = R   (B) $Y = [1, \infty)$
(C) $Y = [4, \infty)$ (D) $Y = [5, \infty)$

Q.20 Let $f : R \to R$ be a function defined by $f(x) = x^3 + x^2 + 3x + \sin x$. Then f is :
(A) one – one & onto   (B) one – one & into
(C) many one & onto   (D) many one & into

Q.21 Which of the following function from $A = \{x : -1 \le x \le 1\}$ to itself are bijections-
(A) $f(x) = \dfrac{x}{2}$ (B) $g(x) = \sin\left(\dfrac{\pi x}{2}\right)$
(C) $h(x) = |x|$ (D) $k(x) = x^2$

Q.22 If $f : \left[\dfrac{\pi}{4} - \dfrac{1}{2}, \dfrac{3\pi}{4} - \dfrac{1}{2}\right] \to [-1, 1]$ is defined by $f(x) = \sin(2x + 1)$, then f is
(A) one one into   (B) many one onto
(C) one one onto   (D) many one into

Q.23 The number of bijective functions from set A to itself when A contains 106 elements is
(A) 106   (B) 106!
(C) 106^{106} (D) $106^{106} - 106!$

Question based on Inverse function

Q.24 If $f(x) = x^3 - 1$ and domain of $f = \{0, 1, 2, 3\}$, then domain of f^{-1} is -
(A) {0, 1, 2, 3}   (B) {1, 0, –7, –26}
(C) {–1, 0, 7, 26}   (D) {0, –1, –2, –3}

Q.25 The inverse of the function $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ is
(A) $\dfrac{1}{2}\log\dfrac{1+x}{1-x}$ (B) $\dfrac{1}{2}\log\dfrac{2+x}{2-x}$
(C) $\dfrac{1}{2}\log\dfrac{1-x}{1+x}$ (D) $2\log(1 + x)$

Question based on Composite function

Q.26 The function $f(x)$ is defined in [0, 1] then the domain of definition of the function $f[\ell n(1 - x^2)]$ is given by :
(A) $x \in \{0\}$
(B) $x \in [-\sqrt{1+e} - 1] \cup [1 + \sqrt{1+e}]$
(C) $x \in (-\infty, \infty)$
(D) None of these

Q.27 If $f : R \to R$, $f(x) = x^3 + 3$, and $g : R \to R$, $g(x) = 2x + 1$, then $f^{-1}og^{-1}(23)$ equals-
(A) 2   (B) 3
(C) $(14)^{1/3}$ (D) $(15)^{1/3}$

Q.28 If $f(x) = e^{3x}$ and $g(x) = \ell n\, x$, $x > 0$, then $(fog)(x)$ is equal to-
(A) $3x$ (B) x^3
(C) $\log 3x$ (D) $3 \log x$

Q.29 If $f(x) = x^3 - x$ and $g(x) = \sin 2x$, then-
(A) $g[f(1)] = 1$ (B) $f\left(g\left(\dfrac{\pi}{12}\right)\right) = -\dfrac{3}{8}$
(C) $g\{f(2)\} = \sin 2$ (D) None of these

Question base don Periodic function

Q.30 If $f : R \to R$ is a function satisfying the property $f(x+1) + f(x+3) = 2\ \forall\ x \in R$ then the period (may not be fundamental period) of $f(x)$ is
(A) 3   (B) 4   (C) 7   (D) 6

Q.31 The fundamental period of the function:
$f(x) = x + a - [x + b] + \sin \pi x + \cos 2\pi x + \sin 3\pi x + \cos 4\pi x + \ldots + \sin(2i - 1)\pi x + \cos 2n\pi x$ for every $a, b \in R$ is:
(where [.] denotes the greatest integer function)
(A) 2   (B) 4   (C) 1   (D) 0

Q.32 Let $f(x) = \sin\sqrt{[a]}\,x$ (where [ ] denotes the greatest integers function). If f is periodic with fundamental period π, then a belongs to -
(A) [2, 3] (B) {4, 5}
(C) [4, 5] (D) [4, 5)

Q.33 The function $f(x) = \left|\cos^5\left(\frac{x}{2}\right)\right|$ is periodic with fundamental period
(A) π (B) 2π
(C) $\frac{\pi}{2}$ (D) 4π

Q.34 The fundamental period of
$f(x) = \cos(\sin x) + \cos(\cos x)$ is
(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$
(C) π (D) $\frac{\pi}{2}$

Q.35 Fundamental period of the function
$f(x) = |\sin \pi x| + e^{3(x - [x])}$ (where [ ] represent greatest integer function) is-
(A) 1 (B) 2
(C) $\frac{1}{3}$ (D) None of these

Question based on Even and odd function

Q.36 Which of the following is an even function?
(A) $x\,\frac{a^x - 1}{a^x + 1}$ (B) $\tan x$
(C) $\frac{a^x - a^{-x}}{2}$ (D) $\frac{a^x + 1}{a^x - 1}$

Q.37 Which of the following function is an odd function
(A) $f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x + x^2}$
(B) $f(x) = x\left(\frac{a^x + 1}{a^x - 1}\right)$
(C) $f(x) = \log\left(\frac{1 - x}{1 + x^2}\right)$
(D) $f(x) = k$ (constant)

Question based on Miscellaneous points

Q.38 The set of points for which
$f(x) = \cos(\sin x) > 0$ contains -
(A) $(-\infty, 0]$ (B) $[-1, 1]$
(C) $(-\infty, \infty)$ (D) All are correct

Q.39 If $[x]$ stands for the greatest integer function, then the value of
$$\left[\frac{1}{2} + \frac{1}{1000}\right] + \left[\frac{1}{2} + \frac{2}{1000}\right] + \ldots + \left[\frac{1}{2} + \frac{999}{1000}\right]$$
(A) 498 (B) 499
(C) 500 (D) 501

Q.40 Let the function $f(x) = 3x^2 - 4x + 8\log(1 + |x|)$ be defined on the interval [0, 1]. The even extension of $f(x)$ to the interval [–1, 0] is -
(A) $3x^2 + 4x + 8\log(1 + |x|)$
(B) $3x^2 - 4x + 8\log(1 + |x|)$
(C) $3x^2 + 4x - 8\log(1 + |x|)$
(D) $3x^2 - 4x - 8\log(1 + |x|)$

Q.41 Let $f : N \to N$ where $f(x) = x + (-1)^{x-1}$ then f is-
(A) Inverse of itself (B) Even function
(C) Periodic (D) Identity

EXERCISE (Level-2)

Single correct answer type questions

Q.1 If fundamental period of $\dfrac{\cos(\sin nx)}{\tan(x/n)}$ $(n \in N)$ is 6π then n is equal to
(A) 3 (B) 2 (C) 6 (D) 1

Q.2 Let $f(x) = \sin^2\left(\dfrac{x}{2}\right) + \cos^2\left(\dfrac{x}{2}\right)$ and $g(x) = \sec^2 x - \tan^2 x$. The two functions are equal over the set -
(A) ϕ
(B) $R - \left\{x : x = (2n+1)\dfrac{\pi}{2}, n \in Z\right\}$
(C) R
(D) None of these

Q.3 Domain and range of $\sin\left(\log\left(\dfrac{\sqrt{4-x^2}}{1-x}\right)\right)$ is
(A) $[-2, 1), (-1, 1)$ (B) $(-2, 1), [-1, 1]$
(C) $(-2, 1)$, R (D) None of these

Q.4 The range of $\sin^{-1}\left[x^2 + \dfrac{1}{2}\right] + \cos^{-1}\left[x^2 - \dfrac{1}{2}\right]$ where [ ] represent greatest integer function
(A) $\left\{\dfrac{\pi}{2}, \pi\right\}$ (B) $\{\pi\}$
(C) $\left\{\dfrac{\pi}{2}\right\}$ (D) None of these

Q.5 Let $f(x) = \dfrac{9^x}{9^x + 3}$ and $f(x) + f(1-x) = 1$ then find value of $f\left(\dfrac{1}{1996}\right) + \left(\dfrac{2}{1996}\right) + \ldots + f\left(\dfrac{1995}{1996}\right)$ is-
(A) 998 (B) 997
(C) 997.5 (D) 998.5

Q.6 If $f(x)$ be a polynomial satisfying $f(x).\ f\left(\dfrac{1}{x}\right) = f(x) + f\left(\dfrac{1}{x}\right)$ and $f(4) = 65$ then $f(6) = ?$
(A) 176 (B) 217
(C) 289 (D) None of these

Q.7 Let $f : R \to R$ defined by $f(x) = \dfrac{\sin([x]\pi)}{x^2 + 2x + 4}$, where [ ] represent greatest integer function, then which one is not true -
(A) f is periodic (B) f is even
(C) f is many-one (D) f is onto

Q.8 Let $f : R \to R$ be a function defined by $f(x) = x + \sqrt{x^2}$, then f is-
(A) Injective (B) Surjective
(C) Bijective (D) None of these

Q.9 Which of the following functions are equal?
(A) $f(x) = x$, $g(x) = \sqrt{x^2}$
(B) $f(x) = \log x^2$, $g(x) = 2\log x$
(C) $f(x) = 1$, $g(x) = \sin^2 x + \cos^2 x$
(D) $f(x) = \dfrac{x}{x}$, $g(x) = 1$

Q.10 Let $f : (4, 6) \to (6, 8)$ be a function defined by $f(x) = x + \left[\dfrac{x}{2}\right]$, where [ ] represent greatest integer function then $f^{-1}(i)$ is equal to -
(A) $x - 2$ (B) $x - \left[\dfrac{x}{2}\right]$
(C) $-x - 2$ (D) None of these

Q.11 The interval for which $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \dfrac{\pi}{2}$ holds-
(A) $[0, \infty)$ (B) $[0, 3]$
(C) $[0, 1]$ (D) $[0, 2]$

Q.12 The function $f(x) = \sqrt{\log_{10}\cos(2\pi x)}$ exists -
(A) for any rational x
(B) only when x is a positive integer
(C) only when x is fractional
(D) for any integer value of x including zero

Q.13 The domain of the function $\sec^{-1}[x^2 - x + 1]$, is given by (where [·] is greatest integer function)-
(A) $[0, 1]$ (B) $(-\infty, 0] \cup [1, \infty)$
(C) $\left[\dfrac{1-\sqrt{5}}{2}, \dfrac{1+\sqrt{5}}{2}\right]$ (D) None of these

Q.14 The domain of definition of the function $f(x) = \dfrac{\cot^{-1} x}{\sqrt{\{x^2 - [x^2]\}}}$, where $[x]$ denotes the greatest integer less than or equal to x is -
(A) R
(B) $R - \{\pm\sqrt{n} : n \in I^+ \cup \{0\}\}$
(C) $R - \{0\}$
(D) $R - \{n : n \in I\}$

Q.15 The domain of the definition of $f(x) = \log\{(\log x)^2 - 5\log x + 6\}$ is equal to-
(A) $(0, 10^2)$ (B) $(10^3, \infty)$
(C) $(10^2, 10^3)$ (D) $(0, 10^2) \cup (10^3, \infty)$

Q.16 If A = $\left\{x : \frac{\pi}{6} \le x \le \frac{\pi}{3}\right\}$ and $f(x) = \cos x - x(1 + x)$ then $f(A)$ is equal to-

(A) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

(B) $\left[-\frac{\pi}{3}, -\frac{\pi}{6}\right]$

(C) $\left[\frac{1}{2} - \frac{\pi}{3}\left(1 + \frac{\pi}{3}\right), \frac{\sqrt{3}}{2} - \frac{\pi}{6}\left(1 + \frac{\pi}{6}\right)\right]$

(D) $\left[\frac{1}{2} + \frac{\pi}{3}\left(1 - \frac{\pi}{3}\right), \frac{\sqrt{3}}{2} + \frac{\pi}{6}\left(1 - \frac{\pi}{6}\right)\right]$

Q.17 If A be the set of all triangles and B that of positive real numbers, then the mapping $f : A \to B$ given by $f(\Delta)$ = area of Δ, ($\Delta \in$ A) is
(A) one-one into mapping
(B) one-one onto mapping
(C) many-one into mapping
(D) many-one onto mapping

Q.18 Let $f : R \to A = \left\{y \mid 0 \le y < \frac{\pi}{2}\right\}$ be a function such that $f(x) = \tan^{-1}(x^2 + x + k)$, where k is a constant. The value of k for which f is an onto function, is -
(A) 1 (B) 0 (C) $\frac{1}{4}$ (D) None

Q.19 Which of the following functions are not injective mapping-
(A) $f(x) = |x + 1|$, $x \in [-1, \infty)$
(B) $g(x) = x + \frac{1}{x}$; $x \in (0, \infty)$
(C) $h(x) = x^2 + 4x - 5$; $x \in (0, \infty)$
(D) $k(x) = e^{-x}$; $x \in [0, \infty)$

Q.20 Let f be an injective map. with domain $\{x, y, z\}$ and range {1, 2, 3}, such that exactly one of the following statements is correct and the remaining are false : $f(x) = 1$, $f(y) \ne 1$, $f(z) \ne 2$. The value of $f^{-1}(1)$ is -
(A) x (B) y (C) z (D) None

Q.21 Let $f : R \to R$ and $g : R \to R$ be two one-one onto functions such that they are mirror image of each other about the line $y = 0$, then $h(x) = f(x) + g(x)$ is-
(A) One-one and onto
(B) One-one but not onto
(C) Not one-one but onto
(D) Neither one-one nor onto

Q.22 Fundamental period of $f(x) = e^{\cos\{x\}} + \sin \pi[x]$ is (where [.] and { } denote the greatest integer function and fractional part of function respectively).
(A) 1 (B) 2 (C) π (D) 2π

Q.23 If $f(x) = \cos(ax) + \sin(bx)$ is periodic, then which of the followings is false -
(A) a and b both are rational
(B) non-periodic if a is rational but b is irrational
(C) non-periodic if a is irrational but b is rational
(D) None of these

Q.24 If $f : [-20, 20] \to R$ is defined by $f(x) = \left[\frac{x^2}{a}\right] \sin x + \cos x$, is an even function, then the set of values of a is (Where [.] denotes greatest integer function)-
(A) $(-\infty, 100)$ (B) $(400, \infty)$
(C) $(-400, 400)$ (D) None of these

Q.25 Let f be a function satisfying $f(x + y) = f(x).f(y)$ for all $x, y \in R$. If $f(1) = 3$ then $\sum_{r=1}^{n} f(r)$ is equal to-
(A) $\frac{3}{2}(3^n - 1)$ (B) $\frac{3}{2}n(n + 1)$
(C) $3^{n+1} - 3$ (D) None of these

Q.26 If $f(x) = [x^2] - [x]^2$ where [·] denotes the greatest integer function and $x \in [0, 2]$, the set of values of $f(x)$ is -
(A) {–1, 0} (B) {–1, 0, 1} (C) {0} (D) {0, 1, 2}

Q.27 Domain of definition of the function
$f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$, is-
(A) $(-1, 0) \cup (1, 2) \cup (2, \infty)$ (B) (1, 2)
(C) $(-1, 0) \cup (1, 2)$ (D) $(1, 2) \cup (2, \infty)$

Q.28 If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is-
(A) $\frac{7n(n+1)}{2}$ (B) $\frac{7n}{2}$
(C) $\frac{7(n+1)}{2}$ (D) $7n(n+1)$

Q.29 Which of the following functions is inverse of itself -
(A) $f(x) = \frac{1-x}{1+x}$ (B) $g(x) = 5^{\log x}$
(C) $h(x) = 2^{x(x-1)}$ (D) None of these

Q.30 If $f(\theta) = \frac{(2\cos\theta - 1)(2\cos 2\theta - 1)(2\cos 4\theta - 1).....(2\cos 2^{n-1}\theta - 1)}{2\cos 2^n\theta + 1}$
for $n \in$ N and $\theta \ne 2m\pi \pm \frac{2\pi}{3}$, $m \in$ I, then $f\left(\frac{\pi}{4}\right)$ =
(A) $1 - \sqrt{2}$ (B) $\sqrt{2} - 1$
(C) $\sqrt{2} + 1$ (D) None of the

EXERCISE (Level-3)

Part-A : Multiple correct answer type questions

Q.1 If $f(x) = \sqrt{x^2 - |x|}$, $g(x) = \dfrac{1}{\sqrt{9-x^2}}$ then D_{f+g} contains
(A) (–3, –1) (B) [1, 3)
(C) [–3, 3] (D) $\{0\} \cup [1, 3)$

Q.2 If $f(x) = \dfrac{3x-1}{3x^3+2x^2-x}$ and $S = \{x \mid f(x) > 0\}$ then S contains
(A) $(-\infty, -2)$ (B) $\left(\dfrac{1}{3}, 5\right)$
(C) $(-\infty, -1)$ (D) $(0, \infty) - \left\{\dfrac{1}{3}\right\}$

Q.3 If D is the domain of the function
$f(x) = \sqrt{1-2x} + 3\sin^{-1}\left(\dfrac{3x-1}{2}\right)$ then D contains-
(A) $\left[-\dfrac{1}{3}, \dfrac{1}{2}\right]$ (B) $\left[-\dfrac{1}{3}, 0\right]$
(C) $\left[-\dfrac{1}{3}, 1\right]$ (D) $\left[\dfrac{1}{2}, 1\right]$

Q.4 Let $A = R - \{2\}$ and $B = R - \{1\}$.
Let $f : A \to B$ be defined by $f(x) = \dfrac{x-3}{x-2}$ then–
(A) f is one-one (B) f is onto
(C) f is bijective (D) None of these

Q.5 If $F(x) = \dfrac{\sin \pi[x]}{\{x\}}$, then F(x) is:
(A) Periodic with fundamental period 1
(B) Even
(C) Range is singleton
(D) Identical to $\text{sgn}\left(\text{sgn}\dfrac{\{x\}}{\sqrt{\{x\}}}\right) - 1$, where $\{x\}$ denotes fractional part function and [.] denotes greatest integer function and sgn (x) is a signum function.

Q.6 Let $f : [-1, 1] \to [0, 2]$ be a linear function which is onto then $f(x)$ is/are
(A) $1 - x$ (B) $1 + x$ (C) $x - 1$ (D) $x + 2$

Q.7 In the following functions defined from [–1, 1] to [–1, 1] the functions which are not bijective are:
(A) $\sin(\sin^{-1}x)$ (B) $\dfrac{2}{\pi}\sin^{-1}(\sin x)$
(C) $(\text{sgn } x)\, \ell n\, e^x$ (D) $x^3 \text{ sgn } x$

Q.8 Which of the following function is periodic ?
(A) $\text{sgn}(e^{-x})$
(B) $\sin x + |\sin x|$
(C) $\min(\sin x, |x|)$
(D) $\left[x + \dfrac{1}{2}\right] + \left[x - \dfrac{1}{2}\right] + 2[-x]$
Where $[x]$ denotes greatest integer function.

Q.9 If $f(x) = \begin{cases} 2x+3 & x \le 1 \\ a^2x+1 & x > 1 \end{cases}$ then values of 'a' for which $f(x)$ is injective is
(A) –3 (B) 3 (C) 0 (D) 1

Q.10 Consider the function $y = f(x)$ satisfying the condition $f\left(x + \dfrac{1}{x}\right) = x^2 + \dfrac{1}{x^2}$ $(x \neq 0)$, then
(A) domain of $f(x)$ is R
(B) domain of $f(x)$ is R – (–2, 2)
(C) range of $f(x)$ is $[-2, \infty)$
(D) range of $f(x)$ is $[2, \infty)$

Q.11 Consider the real-valued function satisfying $2f(\sin x) + f(\cos x) = x$. Then
(A) domain of $f(x)$ is R
(B) domain of $f(x)$ is [–1, 1]
(C) range of $f(x)$ is $\left[-\dfrac{2\pi}{3}, \dfrac{\pi}{3}\right]$
(D) range of $f(x)$ is R

Q.12 Let $f(x) = x^2 - 2ax + a(a+1)$, $f : [a, \infty) \to [a, \infty)$. If one of the solutions of the equation $f(x) = f^{-1}(x)$ is 5049, then the other may be
(A) 5051 (B) 5048 (C) 5052 (D) 5050

Q.13 If $f : R^+ \to R^+$ is a polynomial function satisfying the functional equation
$f(f(x)) = 6x - f(x)$, then $f(17)$ is equal to -
(A) 17 (B) –51 (C) 34 (D) –34

Q.14 $f : R \to [-1, \infty)$ and $f(x) = \ell n([|\sin 2x| + |\cos 2x|])$ (where [.] is the greatest integer function)
(A) $f(x)$ has range Z
(B) $f(x)$ is periodic with fundamental period $\pi/4$
(C) $f(x)$ is invertible in $\left[0, \dfrac{\pi}{4}\right]$
(D) $f(x)$ is into function

Q.15 Let $f(x) = \text{sgn}(\cot^{-1}x) + \tan\left(\dfrac{\pi}{2}[x]\right)$, where $[x]$ is the greatest integer function less than or equal x. Then which of the following alternatives is/are true ?
(A) $f(x)$ is many one but not even function
(B) $f(x)$ is periodic function
(C) $f(x)$ is bounded function
(D) Graph of $f(x)$ remains above the x-axis

Q.16 Let $f : R \to R$ be defined by
$f(x) = \max(1 + |x|, 2 - |x|)$ then which of the following holds(s) good ?
(A) f is periodic function
(B) f is neither injective nor surjective
(C) f is even function
(D) Range of $f = \left[\frac{3}{2}, \infty\right)$

Q.17 Which of the following pair(s) of function have same graphs ?
(A) $f(x) = \frac{\sec x}{\cos x} - \frac{\tan x}{\cot x}$, $g(x) = \frac{\cos x}{\sec x} + \frac{\sin x}{\operatorname{cosec} x}$
(B) $f(x) = \text{sgn}(x^2 - 4x + 5)$,
$g(x) = \text{sgn}\left(\cos^2 x + \sin^2\left(x + \frac{\pi}{3}\right)\right)$ where sgn denotes signum function
(C) $f(x) = e^{\ell n(x^2+3x+3)}$, $g(x) = x^2 + 3x + 3$
(D) $f(x) = \frac{\sin x}{\sec x} + \frac{\cos x}{\operatorname{cosec} x}$, $g(x) = \frac{2\cos^2 x}{\cot x}$

Q.18 Let f be a constant function with domain R and g be a certain function with domain R. Two ordered pairs in f are $(4, a^2 - 5)$ and $(2, 4a - 9)$ for some real number a. Also domain of $\frac{f}{g}$ is R – {7}. Then
(A) $a = 2$
(B) $(f(10))^{100} = 1$
(C) $(100)^{g(7)} = 1$
(D) $\int_0^1 f(x)\,dx = 1$

Part-B : Assertion Reason type Questions

The following questions 19 to 22 consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.
(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.
(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.
(C) If Assertion is true but the Reason is false.
(D) If Assertion is false but Reason is true

Q.19 **Assertion** : Function $f(x) = \sin x + \{x\}$ is periodic with period 2π.
Reason : $\sin x$ and $\{x\}$ are both periodic functions with period 2π and 1 respectively.

Q.20 **Assertion:** If $f(x)$ & $g(x)$ both are one-one, then $f(g(x))$ is also one-one.
Reason : If, $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$ then $f(x)$ is one-one

Q.21 **Assertion:** Let $f: [0, 3] \to [1, 13]$ is defined by $f(x) = x^2 + x + 1$ then inverse is
$f^{-1}(x) = \frac{-1 + \sqrt{4x - 3}}{2}$
Reason: Many-one function is not invertible

Q.22 **Assertion:** Fundamental period of $\cos x + \cot x$ is 2π.
Reason: If the period of $f(x)$ is T_1 and the period of $g(x)$ is T_2, then the fundamental period of $f(x) + g(x)$ is the L.C.M. of T_1 and T_2.

Part-C : Column Matching type Questions

Match the entry in Column 1 with the entry in Column 2.

Q.23 Match the following :

Column 1	Column 2
(A) Range of $\sqrt{[\sin 2x] - [\cos 2x]}$	(P) {1, 2, 3}
(B) Domain of $\sqrt{{}^{x^2+4x}C_{2x^2+3}}$	(Q) {1}
(C) Range of $\sqrt{\log(\cos(\sin x))}$	(R) {0, 1}
(D) Range of $[\,|\sin x| + |\cos x|\,]$	(S) {0}

(Where [.] denotes greatest integer function)

Q.24 Match the following :

Column 1	Column 2
(A) Period of $\frac{1}{2}\left\{\frac{\|\sin x\|}{\cos x} + \frac{\|\cos x\|}{\sin x}\right\}$	(P) 2
(B) Range of $\cos^{-1}\sqrt{\log_{[x]}\frac{\|x\|}{x}}$	(Q) 2π
(C) Total number of solution of $x^2 - 4 - [x] = 0$ is	(R) 1
(D) Fundamental period of $e^{\cos^4 \pi x + x - [x] + \cos^2 \pi x}$	(S) $\frac{\pi}{2}$

(Where [.] denotes greatest integer function)

Q.25 Match the following :

Column 1	Column 2
(A) Domain of $f(x) = \sqrt{2^x - 3^x} + \log_3 \log_{1/2} x$ is	(P) [0, 1]
(B) Solution set of equation $2\cos^2 \frac{x}{2} \sin^2 x = x^2 + \frac{1}{x^2}$ is	(Q) [0, ∞)
(C) If $A = \left\{(x, y);\ y = \frac{1}{x},\ x \in R_0\right\}$ & $B = \{(x, y); y = -x, x \in R\}$ then $A \cap B$ is	(R) [1, ∞)
(D) The functions $f(x) = \sqrt{x}\,\sqrt{x-1}$ & $\phi(x) = \sqrt{x^2 - x}$ are identical in	(S) ϕ

Q.26 Match the following :

Column 1	Column 2
(A) The fundamental period of the function $y = \sin\left(2\pi t + \frac{\pi}{3}\right) + 2\sin\left(3\pi t + \frac{\pi}{4}\right) + 3\sin 5\pi t$ is	(P) $\frac{1}{2}$
(B) $y = \{\sin(\pi x)\}$ is a many one function for $x \in (0, a)$ where { } denotes fractional part of x and a may be	(Q) 8
(C) The Fundamental period of the function $y = \frac{1}{2}\left(\frac{\lvert\sin(\pi/4)x\rvert}{\cos(\pi/4)x} + \frac{\sin(\pi/4)x}{\lvert\cos(\pi/4)x\rvert}\right)$	(R) 2
(D) If $f : [0, 2] \to [0, 2]$ is bijective function defined by $f(x) = ax^2 + bx + c$, where a, b, c are non - zero real numbers, then $f(2)$ is equal to	(S) 0

Part-D : Passage based objective questions

Passage # 1 (Q.27 to 29)

Let $f(x) = x^2 - 3x + 2$, $g(x) = f(\lvert x\rvert)$

$h(x) = \lvert g(x)\rvert$ and $I(x) = \lvert g(x)\rvert - [x]$

are four function, where $[x]$ is the integral part of real x.

Q.27 Find the value of 'a' such that equation $g(x) - a = 0$ has exactly 3 real roots-

(A) 2 (B) 1

(C) 0 (D) None of these

Q.28 Find the set of values of 'b' such that equation $h(x) - b = 0$ has exactly 8 real solution

(A) $b \in \left[0, \frac{1}{4}\right]$ (B) $b \in \left[0, \frac{1}{4}\right)$

(C) $b \in \left(0, \frac{1}{4}\right)$ (D) None of these

Q.29 Which statement is true for $I(x) = 0$ -

(A) Two values of x is satisfied for $I(x) = 0$

(B) One value of x is satisfied for $I(x) = 0$ and that x lie between 1 and 2

(C) One value of x is satisfied for $I(x) = 0$ and that x lie between 3 and 4

(D) None of these

Passage # 2 (Q.30 to 32)

If $f(x) = 0$; if $x \in Q$

$= 1$; if $x \notin Q$.

then answer the following questions-

Q.30 $f(x)$ is -

(A) an even function

(B) an odd function

(C) Neither even nor odd function

(D) one-one function

Q.31 $f(f(x))$ is-

(A) a constant function

(B) an even function

(C) an odd function

(D) many one function

Q.32 Domain of $g(x) = \ln(\text{sgn } f(x))$ is-

(A) R

(B) set of all rational numbers

(C) set of all irrational number

(D) R^+

Passage # 3 (Q.33 to 35)

Consider the function

$$f(x) = \begin{cases} x - [x] - \frac{1}{2}; & if \quad x \notin I \\ 0 \quad ; & if \quad x \in I \end{cases}$$

where [.] denotes greatest integer function.

If $g(x) = \max.\{x^2, f(x), \lvert x\rvert\}$; $-2 \le x \le 2$, then

Q.33 Range of $f(x)$ is-

(A) [0, 1) (B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right)$

Q.34 $f(x)$ is-

(A) non periodic

(B) periodic with fundamental period 1

(C) periodic with fundamental period 2

(D) periodic with fundamental period $\frac{1}{2}$

Q.35 The set of values of a, if $g(x) = a$ has three real and distinct solutions, is -

(A) $\left(0, \frac{1}{2}\right)$ (B) $\left(0, \frac{1}{4}\right)$

(C) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (D) (0, 1)

Passage # 4 (Q.36 to 38)

Consider the function

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x \le 1 \\ \ell n x, & 1 < x \le e \end{cases}$$

Let $f_1(x) = f(\lvert x\rvert)$

$f_2(x) = \lvert f(\lvert x\rvert)\rvert$

$f_3(x) = f(-x)$

Q.36 Number of positive solutions of the equation $2f_2(x) - 1 = 0$ is-

(A) 4 (B) 3 (C) 2 (D) 1

Q.37 Number of integral solution of the equation $f_1(x) = f_2(x)$ is

(A) 1 (B) 2 (C) 3 (D) 4

Q.38 If $f_4(x) = \log_{27}(f_3(x) + 2)$, then range of $f_4(x)$ is

(A) $[1, 9]$ (B) $\left[\frac{1}{3}, \infty\right)$ (C) $\left[0, \frac{1}{3}\right]$ (D) $[1, 27]$

Part-E : Numeric Response Type Questions

Q.39 Let $f(x) = \left[\frac{1}{\cos\{x\}}\right]$ where $[y]$ and $\{y\}$ denote greatest integer and fractional part function respectively and $g(x) = 2x^2 - 3x(k+1) + k(3k+1)$. If $g(f(x)) < 0\ \forall\ x \in R$ then find the number of integral values of k.

Q.40 If $x = \log_4\left(\frac{2f(x)}{1-f(x)}\right)$, then find $(f(2010) + f(-2009))$.

Q.41 If M and m are maximum and minimum value of $f(\theta) = 5\sin^2\theta - 8\sin\theta + 4$, $\theta \in \left[\frac{\pi}{3}, \frac{5\pi}{6}\right]$, respectively then find the value of $(2Mm)$.

Q.42 Let f be a function such that $4f(x^{-1} + 1) + 8f(x + 1) = \log_{12} x$, then find the value of $4(f(10) + f(13) + f(17))$.

Q.43 Let f be a real valued invertible function such that $f\left(\frac{2x-3}{x-2}\right) = 5x - 2$, $x \neq 2$. Find $f^{-1}(13)$.

Part-F : Subjective Type Questions

Q.44 Find the domains of definitions of the following functions:
(Read the symbols [*] and {*} as greatest integers and fractional part functions respectively)

(i) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

(ii) $f(x) = \log_7\log_5\log_3\log_2(2x^3 + 5x^2 - 14x)$

(iii) $f(x) = \ell n\left(\sqrt{x^2 - 5x - 24} - x - 2\right)$

(iv) $f(x) = \sqrt{\frac{1 - 5^x}{7^{-x} - 7}}$

(v) $y = \log_{10}\sin(x - 3) + \sqrt{16 - x^2}$

(vi) $f(x) = \log_{100x}\left(\frac{2\log_{10} x + 1}{-x}\right)$

(vii) $f(x) = \frac{1}{\sqrt{4x^2 - 1}} + \ell n\, x(x^2 - 1)$

(viii) $f(x) = \sqrt{\log_{1/2}\frac{x}{x^2 - 1}}$

(ix) $f(x) = \sqrt{x^2 - |x|} + \frac{1}{\sqrt{9 - x^2}}$

(x) $f(x) = \sqrt{\log_x(\cos 2\pi x)}$

(xi) $f(x) = \frac{\sqrt{\cos x - (1/2)}}{\sqrt{6 + 35x - 6x^2}}$

(xii) $f(x) = \sqrt{\log_{1/3}(\log_4([x]^2 - 5))}$

(xiii) $f(x) = \frac{[x]}{2x - [x]}$

(xiv) $f(x) = \log_x \sin x$

(xv) $f(x) = \log_{\left[x + \frac{1}{x}\right]}|x^2 - x - 6| + {}^{16-x}C_{2x-1} + {}^{20-3x}P_{2x-5}$

Q.45 Find the domain and range of the following functions. (Read the symbols [*] & {*} as greatest integers & fractional part functions respectively)

(i) $y = \log_{\sqrt{5}}(\sqrt{2}(\sin x - \cos x) + 3)$

(ii) $y = \frac{2x}{1 + x^2}$

(iii) $f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 6}$

(iv) $f(x) = \frac{x}{1 + |x|}$

(v) $y = \sqrt{2 - x} + \sqrt{1 + x}$

(vi) $f(x) = \log_{(\text{cosec}\, x - 1)}(2 - [\sin x] - [\sin x]^2)$

(vii) $f(x) = \frac{\sqrt{x + 4} - 3}{x - 5}$

(viii) $\cot^{-1}(2x - x^2)$.

(ix) $f(x) = \log_2(\sqrt{x - 4} + \sqrt{6 - x})$

Q.46 (a) Draw graphs of the following function, where [ ] denotes the greatest integer function.

(i) $f(x) = x + [x]$

(ii) $y = (x)[x]$ where $x = [x] + (x)$ & $x > 0$ and $x \leq 3$

(iii) $y = \text{sgn}[x]$

(iv) $\text{sgn}(x - |x|)$

(b) Identify the pair(s) of functions which are identical? (where $[x]$ denotes greatest integer and $\{x\}$ denotes fractional part function)

(i) $f(x) = \text{sgn}(x^2 - 3x + 4)$ and $g(x) = e^{[\{x\}]}$

(ii) $f(x) = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$ and $g(x) = \tan x$

(iii) $f(x) = \ell n(1 + x) + \ell n(1 - x)$ and $g(x) = \ell n(1 - x^2)$

(iv) $f(x) = \frac{\cos x}{1 - \sin x}$ & $g(x) = \frac{1 + \sin x}{\cos x}$

Q.47 Let f be a function satisfying $2f(xy) = \{f(x)\}y + \{f(y)\}x$ and $f(1) = k \neq 1$.

Prove that $(k - 1)\sum_{n=1}^{n} F(n) = k^{n+1} - k$

Q.48 Determine all functions f satisfying the functional relation
$$f(x) + f\left(\frac{1}{1-x}\right) = \frac{2(1-2x)}{x(1-x)}$$
where x is a real number, $x \neq 0, x \neq 1$.

Q.49 Let $\{x\}$ and $[x]$ denotes the fractional and integral part of a real number x respectively. Solve $4\{x\} = x + [x]$.

Q.50 Let $f(x) = x^2 + kx$; k is a real number. The set of values of k for which the equation $f(x) = 0$ and $f(f(x)) = 0$ have same real solution set.

Q.51 Let $f(x)$ be defined on $[-2, 2]$ and is given by
$$f(x) = \begin{cases} -1 & -2 \le x \le 0 \\ x-1 & 0 < x \le 2 \end{cases} \text{ and}$$
$g(x) = f(|x|) + |f(x)|$. Then find $g(x)$.

Q.52 Solve the equation:
$|2x - 1| = 3[x] + 2\{x\}$ where $[\cdot]$ and $\{\cdot\}$ denotes greatest integer function and fractional part function respectively.

Q.53 Let
$$f(x) = \begin{cases} 3^{x-1} + \dfrac{8}{3} & \text{for } 0 \le x \le 2 \\ 7 + \log_2(x-2) & \text{for } 2 < x \le 4 \\ x^2 - 9x + 21 & \text{for } 4 < x \le 6 \end{cases}.$$
A set 'B' is formed by elements which are 'f ' images of the elements of set A. If $B = \{1, 3, 5, 7\}$, find A. Hence or otherwise state reasons whether it is possible to have a function, $f^{-1} : B \to A$ or not ?

Q.54 If $f(x) \in [1, 2]$ when $x \in R$ and for a fixed positive real number p,
$f(x + p) = 1 + \sqrt{2f(x) - \{f(x)\}^2}$ for all $x \in R$
then prove that $f(x)$ is a periodic function.

Q.55 If $f(a - x) = f(a + x)$ and $f(b - x) = f(b + x)$ for all real x where a, b $(a > b)$ are constants then prove that $f(x)$ is a periodic function.

Q.56 Let $f : R \to R$ be a function given by $f(x + y) + f(x - y) = 2f(x)f(y)$ for all $x, y \in R$ and $f(0) \neq 0$. Prove that $f(x)$ is an even function.

Q.57 Let $f : [-2, 2] \to R$ be a function if, for $x \in [0, 2]$
$$f(x) = \begin{cases} x\tan x, & 0 < x \le \dfrac{\pi}{2} \\ \dfrac{\pi}{2}[x], & \dfrac{\pi}{2} < x \le 2 \end{cases}$$
define f for $x \in [-2, 0]$ when
(i) f is odd function
(ii) f is an even function
(where $[\cdot]$ is the greatest integer function)

Q.58 If $f(x) = -1 + |x - 2|$, $0 \le x < 4$
$g(x) = 2 - |x|$, $-1 \le x \le 3$
Then find $fog(x)$, $gof(x)$ $fof(x)$ & $gog(x)$. Draw rough sketch of the graphs of $fog(x)$ & $gof(x)$.

Q.59 If $f(x) = \ell n(x^2 - x + 2)$; $R^+ \to R$ and
$g(x) = \{x\} + 1$; $[1, 2] \to [1, 2]$,
where $\{x\}$ denotes fractional part of x find the domain and range of $f(g(x))$ when defined.

Q.60 Examine whether the following functions are even or odd or none.

(i) $f(x) = \dfrac{(1+2^x)^7}{2^x}$

(ii) $f(x) = \dfrac{\sec x + x^2 - 9}{x\sin x}$

(iii) $f(x) = \dfrac{x}{e^x - 1} + \dfrac{x}{2} + 1$

(iv) $f(x) = \begin{cases} x|x| & x \le -1 \\ [1+x]+[1-x] & -1 < x < 1 \\ -x|x| & x \ge 1 \end{cases}$

(v) $f(x) = \dfrac{2x(\sin x + \tan x)}{2\left[\dfrac{x+2\pi}{\pi}\right] - 3}$

where [.] denotes greatest integer function.

Q.61 Find the period of the following functions.

(i) $f(x) = 1 - \dfrac{\sin^2 x}{1+\cot x} - \dfrac{\cos^2 x}{1+\tan x}$

(ii) $f(x) = \tan\dfrac{\pi}{2}[x]$ where [.] denotes greatest integer function.

(iii) $f(x) = \ell og(2 + \cos 3x)$

(iv) $f(x) = e^{\ln \sin x} + \tan^3 x - \text{cosec}(3x - 5)$

(v) $f(x) = \sin x + \tan\dfrac{x}{2} + \sin\dfrac{x}{2} + \tan\dfrac{x}{2^2} + \sin\dfrac{x}{2^2} + \tan\dfrac{x}{2^3} \ldots + \sin\dfrac{x}{2^{n-1}} + \tan\dfrac{x}{2^n}$

(vi) $f(x) = \dfrac{\sin x + \sin 3x}{\cos x + \cos 3x}$

Q.62 A function f, defined for all $x, y \in R$ is such that $f(1) = 2$; $f(2) = 8$ and $f(x + y) - kxy = f(x) + 2y^2$, where k is some constant. Find $f(x)$ & show that $f(x+y).f\left(\dfrac{1}{x+y}\right) = k$ for $x + y \neq 0$.

Q.63 Suppose $p(x)$ is a polynomial with integer coefficients. The remainder when $p(x)$ is divided by $x - 1$ is 1 and the remainder when $p(x)$ is divided by $x - 4$ is 10. If $r(x)$ is the remainder when $p(x)$ is divided by $(x - 1)(x - 4)$, find the value of $r(2006)$.

EXERCISE (Level-4)
Old Examination Questions

Section-A [JEE Main]

Q.1 Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval **[AIEEE-2005]**

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left[0, \frac{\pi}{2}\right)$

(C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q.2 A real valued function $f(x)$ satisfies the functional equation
$f(x - y) = f(x)\,f(y) - f(a - x)\,f(a + y)$ where a is a given constant and $f(0) = 1$, then $f(2a - x)$ is equal to - **[AIEEE-2005]**

(A) $-f(x)$ (B) $f(x)$

(C) $f(a) + f(a - x)$ (D) $f(-x)$

Q.3 The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function

$f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$ defined, is-

[AIEEE 2007]

(A) $[0, \pi]$ (B) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(C) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (D) $\left[0, \frac{\pi}{2}\right)$

Q.4 Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = |y \in N : y = 4x + 3$ for some $x \in N|$. Inverse of f is - **[AIEEE 2008]**

(A) $g(y) = 4 + \frac{y+3}{4}$ (B) $g(y) = \frac{y+3}{4}$

(C) $g(y) = \frac{y-3}{4}$ (D) $g(y) = \frac{3y+4}{3}$

Q.5 For real x, let $f(x) = x^3 + 5x + 1$, then - **[AIEEE 2009]**

(A) f is one - one but not onto on R
(B) f is onto on R but not one - one
(C) f is one - one and onto on R
(D) f is neither one - one nor onto on R

Q.6 Let $f(x) = (x + 1)^2 - 1, x \geq -1$
Statement - 1 :
The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.
Statement - 2 :
f is a bijection. **[AIEEE 2009]**

(A) Statement -1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement -1
(B) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement -1.
(C) Statement -1 is true, Statement-2 is false.
(D) Statement -1 is false, Statement-2 is true

Q.7 The domain of the function

$f(x) = \frac{1}{\sqrt{|x| - x}}$ is : **[AIEEE 2011]**

(A) $(-\infty, \infty)$ (B) $(0, \infty)$

(C) $(-\infty, 0)$ (D) $(-\infty, \infty) - \{0\}$

Q.8 Let A and B be nonempty set in R and $f : A \to B$ be a bijective function.
Statement-1: f is an onto function
Statement-2 : There exists a function $g : B \to A$ such that $fog = I_B$.
[AIEEE Online- 2012]

(A) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true

Q.9 The range of the function $f(x) = \frac{x}{1+|x|}$, $x \in R$ is :
[AIEEE Online- 2012]

(A) $[-1, 1]$ (B) R (C) $R - \{0\}$ (D) $(-1, 1)$

Q.10 If P(S) denotes the set of al subsets of a given set S, then the number of one to one functions from the set $S = \{1, 2, 3\}$ to the set P(S) is :
[AIEEE Online- 2012]

(A) 24 (B) 8 (C) 336 (D) 320

Q.11 Let $A = \{1, 2, 3, 4\}$ and $R : A \to A$ be the relation defined by $R = \{(1, 1), (2, 3), (3, 4), (4, 2)\}$. The correct statements is : **[JEE Main Online -2013]**

(A) R does not have an inverse
(B) R is not a one to one function
(C) R is an onto function
(D) R is not a function

Q.12 If $a \in R$ and the equation
$-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval : **[JEE Main -2014]**

(A) $(-\infty, -2) \cup (2, \infty)$ (B) $(-1, 0) \cup (0, 1)$

(C) $(1, 2)$ (D) $(-2, -1)$

Q.13 If $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$, $x \in R$, then the equation $f(x) = 0$ has : **[JEE Main Online -2014]**

(A) no solution
(B) one solution
(C) two solution
(D) more than two solutions

Q.14 Let f be an odd function defined on the set of real numbers such that for $x \geq 0$, $f(x) = 3 \sin x + 4 \cos x$. Then $f(x)$ at $x = -\frac{11\pi}{6}$ is equal to : **[JEE Main Online -2014]**

(A) $\frac{3}{2} + 2\sqrt{3}$ (B) $-\frac{3}{2} + 2\sqrt{3}$
(C) $\frac{3}{2} - 2\sqrt{3}$ (D) $-\frac{3}{2} - 2\sqrt{3}$

Q.15 Let $f : R \to R$ be defined by $f(x) = \frac{|x|-1}{|x|+1}$ then f is **[JEE Main Online -2014]**

(A) Both one – one and onto
(B) One – one but not onto
(C) Onto but not one – one
(D) Neither one – one nor onto

Q.16 Let $f(n) = \left[\frac{1}{3} + \frac{3n}{100}\right]n$, where $[n]$ denotes the greatest integer less than or equal to n. Then $\sum_{n=1}^{56} f(n)$ is equal to: **[JEE Main Online -2014]**

(A) 56 (B) 689 (C) 1287 (D) 1399

Q.17 The function $f(x) = |\sin 4x| + |\cos 2x|$, is a periodic function with period. **[JEE Main Online -2014]**

(A) 2π (B) π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Q.18 If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$, and $S = \{x \in R : f(x) = f(-x)\}$; then S : **[JEE Main - 2016]**

(A) is an empty set
(B) contains exactly one element
(C) contains exactly two elements
(D) contains more than two elements

Q.19 For $x \in \mathbf{R}$, $x \neq 0$, $x \neq 1$, let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x))$, $n = 0, 1, 2, ...$ Then the value of $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$ is equal to - **[JEE Main Online -2016]**

(A) $\frac{8}{3}$ (B) $\frac{5}{3}$ (C) $\frac{4}{3}$ (D) $\frac{1}{3}$

Q.20 The function $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is **[JEE Main - 2017]**

(A) injective but not surjective
(B) surjective but not injective
(C) neither injective nor surjective
(D) invertible

Q.21 Let $f(x) = 2^{10}x + 1$ and $g(x) = 3^{10}x - 1$. If $(fog)(x) = x$, then x is equal to- **[JEE Main Online -2017]**

(A) $\frac{2^{10}-1}{2^{10}-3^{-10}}$ (B) $\frac{1-2^{-10}}{3^{10}-2^{-10}}$
(C) $\frac{3^{10}-1}{3^{10}-2^{-10}}$ (D) $\frac{1-3^{-10}}{2^{10}-3^{-10}}$

Q.22 The function $f : N \to N$ defined by $f(x) = x - 5\left[\frac{x}{5}\right]$, where N is the set of natural numbers and $[x]$ denotes the greatest integer less than or equal to x, is - **[JEE Main Online -2017]**

(A) one-one but not onto
(B) one-one and onto
(C) neither one-one nor onto
(D) onto but not one-one

Q.23 Let $f : A \to B$ be a function defined as $f(x) = \frac{x-1}{x-2}$, where $A = R - \{2\}$ and $B = R - \{1\}$. Then f is - **[JEE-Main Online-2018]**

(A) invertible and $f^{-1}(y) = \frac{2y+1}{y-1}$
(B) invertible and $f^{-1}(y) = \frac{3y-1}{y-1}$
(C) no invertible
(D) invertible and $f^{-1}(y) = \frac{2y-1}{y-1}$

Q.24 For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then J(x) is equal to : **[JEE Main - 2019]**

(A) $\frac{1}{x}f_3(x)$ (B) $f_2(x)$
(C) $f_3(x)$ (D) $f_1(x)$

Q.25 Let A = {x ∈ R : x is not a positive integer}. Define a function $f : A \to R$ as $f(x) = \frac{2x}{x-1}$, then f is : **[JEE Main - 2019]**
(A) not injective
(B) surjective but not injective
(C) injective but not surjective
(D) neither injective nor surjective

Q.26 Let N be the set of natural numbers and two functions f and g be defined as f, g : N → N such that **[JEE Main - 2019]**

$$f(n) = \begin{cases} \frac{n+1}{2} & ; \text{ if } n \text{ is odd} \\ \frac{n}{2} & ; \text{ if } n \text{ is even} \end{cases} \text{; and}$$

$g(n) = n - (-1)^n$. Then fog is
(A) neither one-one nor onto
(B) onto but not one-one
(C) both one-one and onto
(D) one-one but not onto

Q.27 Let $f : R \to R$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$. Then the range of f is **[JEE Main - 2019]**
(A) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (B) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$
(C) $(-1, 1) - \{0\}$ (D) $R - [-1, 1]$

Q.28 Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all x ∈ R, the value of $f_4(x) - f_6(x)$ is equal to **[JEE Main - 2019]**
(A) $\frac{1}{4}$ (B) $\frac{5}{12}$ (C) $\frac{-1}{12}$ (D) $\frac{1}{12}$

Q.29 Let a function $f : (0, \infty) \to (0, \infty)$ be defined by $f(x) = \left|1 - \frac{1}{x}\right|$. Then f is : **[JEE Main - 2019]**
(A) not injective but it is surjective
(B) neiter injective nor surjective
(C) injective only
(D) both injective as well as surjective

Q.30 If $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$, $|x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to - **[JEE Main - 2019]**
(A) $2f(x)$ (B) $2f(x^2)$
(C) $(f(x))^2$ (D) $-2f(x)$

Q.31 Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals **[JEE Main - 2019]**
(A) $2f_1(x+y)\, f_2(x-y)$ (B) $2f_1(x+y)\, f_1(x-y)$
(C) $2f_1(x)\, f_2(y)$ (D) $2f_1(x)\, f_1(y)$

Q.32 Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$, where the function f satisfies $f(x + y) = f(x)f(y)$ for all natural numbers x, y and f(1) = 2. Then the natural number 'a' is : **[JEE Main - 2019]**
(A) 2 (B) 3 (C) 16 (D) 4

Q.33 If the function $f : R - \{1, -1\} \to A$ defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to : **[JEE Main - 2019]**
(A) $R - \{-1\}$ (B) $R - [-1, 0)$
(C) $R - (-1, 0)$ (D) $[0, \infty)$

Q.34 The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is : **[JEE Main - 2019]**
(A) $(1, 2) \cup (2, \infty)$
(B) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$
(C) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
(D) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

Q.35 Let $f(x) = x^2$, x ∈ R. For any A ⊆ R, define g(A) = { x ∈ R : f(x) ∈ A}. If S = [0, 4], then which one of the following statements is not true? **[JEE Main - 2019]**
(A) $g(f(S)) \neq S$ (B) $f(g(S)) = S$
(C) $f(g(S)) \neq f(S)$ (D) $g(f(S)) = g(S)$

Q.36 For $x \in (0, 3/2)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$ If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to : **[JEE Main - 2019]**
(A) $\tan\frac{7\pi}{12}$ (B) $\tan\frac{11\pi}{12}$
(C) $\tan\frac{\pi}{12}$ (D) $\tan\frac{5\pi}{12}$

Q.37 If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to: **[JEE Main - 2020]**
(A) $-\frac{3}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $\frac{1}{2}$

Q.38 Let $f : R \to R$ be a function which satisfies $f(x + y) = f(x) + f(y)\, \forall x, y \in R$. If f(1) = 2 and $g(n) = \sum_{k=1}^{(n-1)} f(k)$, n ∈ N then the value of n, for which g(n) = 20, is : **[JEE Main - 2020]**
(A) 5 (B) 9 (C) 20 (D) 4
Official Ans. by NTA (1)

Q.39 Let f : R → R be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$. Then the composition function $f(g(x))$ is :

[JEE Main - 2021]

(1) onto but not one-one
(2) both one-one and onto
(3) one-one but not onto
(4) neither one-one nor onto

Q.40 Let A = {2, 3, 4, 5, ... , 30} and '$\simeq$' be an equivalence relation on A × A, defined by (a, b) $\simeq$ (c, d), if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

[JEE Main - 2021]

(1) 5 (2) 6 (3) 8 (4) 7

Q.41 Let $f : R - \{3\} \to R - \{1\}$ be defined by $f(x) = \frac{x-2}{x-3}$. Let $g : R \to R$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to

[JEE Main - 2021]

(1) 7 (2) 2 (3) 5 (4) 3

Q.42 Let f : **N** → **R** be a function such that $f(x + y) = 2f(x)\ f(y)$ for natural numbers x and y. If $f(A) = 2$, then the value of α for which $\sum_{k=1}^{10} f(\alpha + k) = \frac{512}{3}(2^{20} - 1)$ holds, is :

[JEE Main-2022]

(A) 2 (B) 3 (C) 4 (D) 6

Q.43 Let $f(x) = \frac{x-1}{x+1}$, $x \in R - \{0, -1, 1\}$. If $f^{n+1}(x) = f(f^{n}(x))$ for all $n \in N$, then $f^{6}(6) = f^{7}(7)$ is equal to **[JEE Main-2022]**

(A) $\frac{7}{6}$ (B) $-\frac{3}{2}$ (C) $\frac{7}{12}$ (D) $-\frac{11}{12}$

Q.44 Let : f : R → R be defined as f(x) = x – 1 and g : R – {1, – 1} → R be defined as

$g(x) = \frac{x^2}{x^2 - 1}$

Then the function fog is: **[JEE Main-2022]**

(A) One-one but not onto
(B) Onto but not one-one
(C) Both one-one and onto
(D) Neither one-one nor onto

Q.45 Let a function f : N → N be defined by

$$f(n) = \begin{cases} 2n, & n = 2, 4, 6, 8.... \\ n - 1, & n = 3, 7, 11, 15.... \\ \frac{n+1}{2} & n = 1, 5, 9, 13.... \end{cases}$$

then, f is **[JEE Main-2022]**

(A) One-one but not onto
(B) Onto but not one-one
(C) Neither one-one nor onto
(D) One-one and onto

Q.46 Let f : **R** → **R** be a continuous function such that f(3x) – f(x) = x. If f(8) = 7, then f(14) is equal to **[JEE Main-2022]**

(A) 4 (B) 10 (C) 11 (D) 16

Q.47 Let f(x) and g(x) be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$ and $g(f(x)) = 4x^2 + 6x + 1$, then the value of f(B) + g(B) is___________.

[JEE Main-2022]

Q.48 The number of functions f, from the set $A = \{x \in N : x^2 - 10x + 9 \le 0\}$ to the set $B = \{n^2 : n \in N\}$ such that $f(x) \le (x - 3)^2 + 1$, for every $x \in A$, is ____ . **[JEE Main-2022]**

Q.49 If $f(x) = \frac{2^{2x}}{2^{2x} + 2}$, $x \in R$, then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + ... + f\left(\frac{2022}{2023}\right)$ is equal to **[JEE Main-2023]**

(A) 1010 (B) 2011 (C) 1011 (D) 2010

Q.50 Let $f(x)$ be a function such that $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in N$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is **[JEE Main-2023]**

(A) 8 (B) 9 (C) 6 (D) 7

Q.51 Let f, g and h be the real valued functions defined of R as

$$f(x)\begin{cases} \frac{x}{|x|}, & x \ne 0 \\ 1, & x = 0 \end{cases}, \quad g(x)\begin{cases} \frac{\sin(x+1)}{(x+1)}, & x \ne -1 \\ 1, & x = -1 \end{cases}$$

and $h(x) = 2[x] - f(x)$, where $[x]$ is the greatest integer $\le x$.

Then the value of $\lim_{x \to 1} g(h(x - 1))$ is

[JEE Main-2023]

(A) 1 (B) –1 (C) sin(1) (D) 0

Q.52 Let $f:(0,1)\to R$ be a function defined by $f(x)=\frac{1}{1-e^{-x}}$, and $g(x)=(f(-x)-f(x))$. Consider two statements
(I) g is an increasing function in (0, 1)
(II) g is one-one in (0, 1)
Then, **[JEE Main-2023]**
(A) Only (I) is true
(B) Both (I) and (II) are true
(C) Only (II) is true
(D) Neither (I) nor (II) is true

Q.53 Let $f : R - \{0, 1\} \to$ be a function such that $f(x)+f\left(\frac{1}{1-x}\right)=1+x$. then f(2) is equal to :
[JEE Main-2023]
(A) $\frac{9}{2}$ (B) $\frac{9}{4}$ (C) $\frac{7}{3}$ (D) $\frac{7}{4}$

Section-B [JEE Advanced]

Q.1 $f(x)=\begin{cases}x, & x\in Q\\ 0, & x\notin Q\end{cases}$; $g(x)=\begin{cases}0 & x\in Q\\ x & x\notin Q\end{cases}$
then $(f-g)$ is **[IIT Scr. 2005]**
(A) one-one, onto
(B) neither one-one, nor onto
(C) one-one but not onto
(D) onto but not one-one

Q.2 If X and Y are two non-empty sets where $f: X\to Y$ is function is defined such that
$f(C)=\{f(x): x\in C\}$ for $C\subseteq X$
and $f^{-1}(D)=\{x: f(x)\in D\}$ for $D\subseteq Y$
for any $A\subseteq X$ and $B\subseteq Y$ then- **[IIT 2005]**
(A) $f^{-1}(f(A))=A$
(B) $f^{-1}(f(A))=A$ only if $f(X)=Y$
(C) $f(f^{-1}(B))=B$ only if $B\subseteq f(x)$
(D) $f(f^{-1}(B))=B$

Q.3 Find the range of values of t for which $2\sin t=\frac{1-2x+5x^2}{3x^2-2x-1}$; $t\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
[IIT 2005]

Q.4 Let $f(x)=\frac{x^2-6x+5}{x^2-5x+6}$ **[IIT 2007]**.

Column-I	Column-II
(A) If $-1<x<1$, then $f(x)$ satisfies	(P) $0<f(x)<1$
(B) If $1<x<2$, then $f(x)$ satisfies	(Q) $f(x)<0$
(C) If $3<x<5$, then $f(x)$ satisfies	(R) $f(x)>0$
(D) If $x>5$, then $f(x)$ satisfies	(S) $f(x)<1$

Q.5 Let $f(x)=x^2$ and $g(x)=\sin x$ for all $x\in R$. Then the set of all x satisfying $(f \circ g \circ g \circ f)(x)=(g \circ g \circ f)(x)$, where $(f \circ g)(x)=f(g(x))$, is
[IIT 2011]
(A) $\pm\sqrt{n\pi}$, $n\in\{0, 1, 2...\}$
(B) $\pm\sqrt{n\pi}$, $n\in\{1, 2...\}$
(C) $\frac{\pi}{2}+2n\pi, n\in\{...., -2, -1, 0, 1, 2, ...\}$
(D) $2n\pi$, $n\in\{...., -2, -1, 0, 1, 2, ...\}$

Q.6 The function $f:[0,3]\to[1,29]$, defined by $f(x)=2x^3-15x^2+36x+1$, is **[IIT 2012]**
(A) one-one and onto.
(B) onto but not one-one.
(C) one-one but not onto.
(D) neither one-one nor onto.

Q.7 Let $f:(-1,1)\to IR$ be such that
$f(\cos 4\theta)=\frac{2}{2-\sec^2\theta}$ for $\theta\in\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{4},\frac{\pi}{2}\right)$.
Then the value(s) of $f\left(\frac{1}{3}\right)$ is (are)
MCQ [IIT 2012]
(A) $1-\sqrt{\frac{3}{2}}$ (B) $1+\sqrt{\frac{3}{2}}$ (C) $1-\sqrt{\frac{2}{3}}$ (D) $1+\sqrt{\frac{2}{3}}$

Q.8 Let $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to R$ be given by
$f(x)=(\log(\sec x+\tan x))^3$. Then
MCQ [IIT-Advance 2014]
(A) $f(x)$ is an odd function
(B) $f(x)$ is a one-one function
(C) $f(x)$ is an onto function
(D) $f(x)$ is an even function

Q.9 Let $f(x)=\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x\in R$ and $g(x)=\frac{\pi}{2}\sin x$ for all $x\in R$. Let $(f \circ g)(x)$ denote $f(g(x))$ and $(g \circ f)(x)$ denote $g(f(x))$. Then which of the following is (are) true ?
MCQ [IIT-Advance 2015]
(A) Range of f is $\left[-\frac{1}{2},\frac{1}{2}\right]$
(B) Range of $f \circ g$ is $\left[-\frac{1}{2},\frac{1}{2}\right]$
(C) $\lim_{x\to 0}\frac{f(x)}{g(x)}=\frac{\pi}{6}$
(D) There is an $x\in R$ such that $(g \circ f)(x)=1$

Q.10 Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is ____. **[JEE - Advance 2018]**

Q.11 Let $E_1 = \left\{x \in R : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$ and

$$E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{is a real number}\right\}$$

$$\left(\begin{array}{l}\text{Here, the inverse trigonometric function} \\ \sin^{-1} x \text{ assumes values in } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].\end{array}\right)$$

Let $f : E_1 \to R$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$ and $g : E_2 \to R$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$

[JEE - Advance 2018]

List-I

(P) The range of f is

(Q) The range of g contains

(R) The domain of f contains

(S) The domain of g is

List-II

(1) $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$

(2) (0, 1)

(3) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(4) $(-\infty, 0) \cup (0, \infty)$

(5) $\left(-\infty, \frac{e}{e-1}\right]$

(6) $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is :

(A) P → 4 ; Q → 2 ; R → 1 ; S → 1

(B) P → 3 ; Q → 3 ; R → 6 ; S → 5

(C) P → 4 ; Q → 2 ; R → 1 ; S → 6

(D) P → 4 ; Q → 3 ; R → 6 ; S → 5

Q.12 If the function f : R → R is defined by $f(x) = |x|(x - \sin x)$, then which of the following statements is **TRUE** ? **[JEE - Advance 2020]**

(A) f is one-one, but **NOT** onto

(B) f is onto, but **NOT** one-one

(C) f is **BOTH** one-one and onto

(D) f is **NEITHER** one-one **NOR** onto

Q.13 Let f : [0, 2] → R be the function defined by $f(x) = (3 - \sin 2\pi x)) \sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right)$.

If $\alpha, \beta \in [0, 2]$ are such that $\{x \in [0, 2] : f(x) \geq 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is ______

[JEE - Advance 2020]

Q.14 Let the function $f : [0, 1] \to R$ be defined by $f(x) = \frac{4^x}{4^x + 2}$. Then the value of

$$f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + f\left(\frac{3}{40}\right) + \ldots\ldots + f\left(\frac{39}{40}\right) - f\left(\frac{1}{2}\right)$$ is ______

[JEE - Advance 2020]

Q.15 Let f : R → R be defined by

$$f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}$$

Then which of the following statements is (are) **TRUE** ? **[JEE -Advance 2021]**

(A) f is decreasing in the interval (–2, –1)

(B) f is increasing in the interval (1, 2)

(C) f is onto

(D) Range of f is $\left[-\frac{3}{2}, 2\right]$

Q.16 Let $f : [0,1] \to [0,1]$ be the function defined by $f(x) = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$. Consider the square region

$S = [0,1] \times [0,1]$. Let $G = \{(x, y) \in S : y > f(x)\}$ be called the green region and $R = \{(x, y) \in S : y < f(x)\}$ be called the red region. Let $L_h = \{(x, h) \in S : x \in [0,1]\}$ be the horizontal line drawn at a height $h \in [0, 1]$. Then which of the following statements is(are) true?

[JEE - Advance 2023]

(A) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the green region below the line L_h

(B) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the red region below the line L_h

(C) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the red region below the line L_h

(D) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the green region below the line L_h

Q.17 Let $f : (0,1) \to \mathbf{R}$ be the function defined as $f(x) = \sqrt{n}$ if $x \in \left[\frac{1}{n+1}, \frac{1}{n}\right)$ where $n \in \mathbf{N}$. Let $g : (0,1) \to \mathbf{R}$ be a function such that $\int_{x^2}^{x} \sqrt{\frac{1-t}{t}}\, dt < g(x) < 2\sqrt{x}$ for all $x \in (0,1)$. Then $\lim_{x \to 0} f(x)g(x)$ **[JEE - Advance 2023]**

(A) does NOT exist (B) is equal to 1
(C) is equal to 2 (D) is equal to 3

Q.18 Let $n \geq 2$ be a natural number and $f : [0, 1] \to R$ be the function defined by

$$f(x) = \begin{cases} n(1-2nx) & \text{if } 0 \le x \le \frac{1}{2n} \\ 2n(2nx-1) & \text{if } \frac{1}{2n} \le x \le \frac{3}{4n} \\ 4n(1-nx) & \text{if } \frac{3}{4n} \le x \le \frac{1}{n} \\ \frac{n}{n-1}(nx-1) & \text{if } \frac{1}{n} \le x \le 1 \end{cases}$$

If n is such that the area of the region bounded by the curves $x = 0$, $x = 1$, $y = 0$ and $y = f(x)$ is 4, then the maximum value of the function f is **[JEE - Advance 2023]**

Q.19 Let $f : [1, \infty) \to R$ be a differentiable function such that $f(1) = \frac{1}{3}$ and $3\int_1^x f(t)dt = xf(x) - \frac{x^3}{3}, x \in [1, \infty)$. Let e denote the base of the natural logarithm. Then the value of f(e) is **[JEE - Advance 2023]**

(A) $\frac{e^2+4}{3}$ (B) $\frac{\log_e 4 + e}{3}$
(C) $\frac{4e^2}{3}$ (D) $\frac{e^2-4}{3}$

Q.20 Let $f: (0,1) \to R$ be the function defined as $f(x) = [4x]\left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right)$, where $[x]$ denotes the greatest integer less than or equal to x. Then which of the following statements is (are) true? **[JEE - Advance 2023]**

(A) The function f is discontinuous exactly at one point in (0, 1)
(B) There is exactly one point in (0, 1) at which the function f is continuous but NOT differentiable
(C) The function f is NOT differentiable at more than three points in (0, 1)
(D) The minimum value of the function f is $-\frac{1}{512}$

EXERCISE (Level-5)
Review Exercise

Q.1 Find the natural number 'a' for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, where the function '$f$' satisfies the relation $f(x + y) = f(x) f(y)$ for all natural numbers x, y and further $f(1) = 2$. **[IIT-1992]**

Q.2 A function $f : R \to R$, where R, is the set of real numbers, is defined by $f(x) = \frac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$
Find the interval of values of α for which $f(x)$ is onto. Is the functions one-to-one for $\alpha = 3$? Justify your answer. **[IIT 1996]**

Q.3 Let $f(x) = [x] \sin\left(\frac{\pi}{[x+1]}\right)$, where [.] denotes the greatest integer function. Then find the domain of f. **[IIT 1996]**

Q.4 If f is an even function defined on the interval $(-5, 5)$, then four real values of x satisfying the equation $f(x) = f\left(\frac{x+1}{x+2}\right)$ are ......,......... and. .... **[IIT-1996]**

Q.5 If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then find the value of $f^{-1}(x)$. **[IIT 99]**

Q.6 Let $[x]$ = the greatest integer less than or equal to x. If all the values of x such that the product $\left[x - \frac{1}{2}\right]\left[x + \frac{1}{2}\right]$ is prime, belongs to the set $[x_1, x_2) \cup [x_3, x_4)$, find the value of $x_1^2 + x_2^2 + x_3^2 + x_4^2$.

Q.7 The set of real values of 'x' satisfying the equality $\left[\frac{3}{x}\right] + \left[\frac{4}{x}\right] = 5$ (where [ ] denotes the greatest integer function) belongs to the interval $\left(a, \frac{b}{c}\right]$ where $a, b, c \in N$ and $\frac{b}{c}$ is in its lowest form. Find the value of $a + b + c + abc$.

Q.8 Let $f : R \to R - \{3\}$ be a function with the property that there exist $T > 0$ such that $f(x + T) = \frac{f(x) - 5}{f(x) - 3}$ for every $x \in R$. Prove that $f(x)$ is periodic.

Q.9 In a function
$$2f(x) + x f\left(\frac{1}{x}\right) - 2f\left(\left|\sqrt{2}\sin\left(\pi\left(x + \frac{1}{4}\right)\right)\right|\right) = 4\cos^2\frac{\pi x}{2} + x\cos\frac{\pi}{x}.$$ Prove that

(i) $f(2) + f\left(\frac{1}{2}\right) = 1$

(ii) $f(2) + f(1) = 0$

Q.10 Verify if $f(x) = \frac{x^2 - 8x + 18}{x^2 + 4x + 30}$ is an one-one function.

Q.11 Find the domain of the function,
$$f(x) = \frac{1}{[|x-1|] + [|7-x|] - 6}$$
Where$[\cdot]$ is greatest integer function.

Q.12 Let n be a positive integer and define
$f(n) = 1! + 2! + 3! + \ldots\ldots + n!$,
where $n! = n(n-1)(n-2) \ldots. 3.2.1$.
Find the polynomial $P(x)$ and $Q(x)$ such that $f(n+2) = P(n) f(n+1) + Q(n) f(n)$, for all $n \geq 1$.

Q.13 Find the domain of
$$y = \sqrt{-\log_{\frac{x+4}{2}}\left(\log_2 \frac{2x-1}{3+x}\right)}$$

Passage (Q.14 & 15)

If notation $[x]$ denotes least integer greater than or equal to x and (.) denotes greatest integer less than or equal to x, then

Q.14 The solution set of the equation $(x)^2 + [x]^2 = [x - 1]^2 + (x + 1)^2$ is -

(A) $\{x; x \in R\}$ (B) $\{x; x \in R - Z\}$

(C) $\{x; x \in Z\}$ (D) $\{x; x \in \phi\}$

Q.15 Let $f(x) = x + (x)$; $x < 0$

$3x - 2(x)$; $x \geq 0$

Range of sgn $f(x)$ is -

(A) $\{-1, 0, 1\}$ (B) $\{-1, 1\}$

(C) $\{1, 0\}$ (D) $\{-1, 0\}$

Q.16 Let A be a set of n distinct elements. Then the total number of distinct functions from A to A is .... & out of these .... are onto functions.

[IIT-1985]

ANSWER KEY

EXERCISE (Level-1)

1. (A) **2.** (D) **3.** (C) **4.** (D) **5.** (A) **6.** (C) **7.** (B)
8. (C) **9.** (C) **10.** (B) **11.** (C) **12.** (A) **13.** (D) **14.** (C)
15. (A) **16.** (D) **17.** (B) **18.** (D) **19.** (B) **20.** (A) **21.** (B)
22. (C) **23.** (B) **24.** (C) **25.** (A) **26.** (A) **27.** (A) **28.** (B)
29. (B) **30.** (B) **31.** (A) **32.** (D) **33.** (B) **34.** (D) **35.** (A)
36. (A) **37.** (A) **38.** (D) **39.** (C) **40.** (A) **41.** (A)

EXERCISE (Level-2)

1. (C) **2.** (B) **3.** (B) **4.** (B) **5.** (C) **6.** (B) **7.** (D)
8. (D) **9.** (C) **10.** (A) **11.** (C) **12.** (D) **13.** (B) **14.** (B)
15. (D) **16.** (C) **17.** (D) **18.** (C) **19.** (B) **20.** (B) **21.** (D)
22. (A) **23.** (D) **24.** (B) **25.** (A) **26.** (D) **27.** (A) **28.** (A)
29. (A) **30.** (B)

EXERCISE (Level-3)

Part-A

1. (A,B,D) **2.** (A,B,C,D) **3.** (A,B) **4.** (A,B,C) **5.** (A,B,C,D) **6.** (A,B) **7.** (B,C,D)
8. (A,B,C,D) **9.** (A,B) **10.** (B,D) **11.** (B,C) **12.** (B,D) **13.** (C) **14.** (B,D)
15. (A,B,C,D) **16.** (B,C,D) **17.** (A,B,C,D) **18.** (A,B,C)

Part-B

19. (D) **20.** (A) **21.** (B) **22.** (C)

Part-C

23. $A \to R, B \to P, C \to S, D \to Q$ **24.** $A \to Q, B \to S, C \to P, D \to R$
25. $A \to S, B \to S, C \to S, D \to R$ **26.** $A \to R, B \to Q,R, C \to Q, D \to S$

Part-D

27. (A) **28.** (C) **29.** (C) **30.** (A) **31.** (A,B,C,D) **32.** (C) **33.** (C)
34. (B) **35.** (C) **36.** (C) **37.** (D) **38.** (C)

Part-E

39. 1 **40.** 1 **41.** 2 **42.** 3 **43.** 3

Part-F

44. (i) $\left[-\frac{5\pi}{4}, \frac{-3\pi}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$ (ii) $\left(-4, -\frac{1}{2}\right) \cup (2, \infty)$ (iii) $(-\infty, -3]$ (iii) $(-\infty, -3]$

(iv) $(-\infty, -1) \cup [0, \infty)$ (v) $(3 - 2\pi < x < 3 - \pi) \cup (3 < x \le 4)$

(vi) $\left(0, \frac{1}{100}\right) \cup \left(\frac{1}{100}, \frac{1}{\sqrt{10}}\right)$ (vii) $(-1 < x < -1/2) \cup (x > 1)$ (vii) $\left[\frac{1-\sqrt{5}}{2}, 0\right) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$

(ix) $(-3, -1] \cup \{0\} \cup [1, 3)$ (x) $\left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right) \cup \{x : x \in N, x \ge 2\}$ (xi) $\left(-\frac{1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$

(xii) $[-3, -2) \cup [3, 4)$ (xiii) $R - \left\{-\frac{1}{2}, 0\right\}$

(xiv) $2n\pi < x < (2n+1)\pi$ but $x \ne 1$ where n is non-negative integer. (xv) $x \in \{4, 5\}$

45. (i) D : $x \in R$ R : $[0, 2]$ (ii) D = R ; range $[-1, 1]$

(iii) D : $\{x \mid x \in R; x \ne -3 ; x \ne 2\}$ R : $\{f(x) \mid f(x) \in R, f(x) \ne 1/5 ; f(x) \ne 1\}$

(iv) D : R ; R : $(-1, 1)$ (v) D : $-1 \le x \le 2$; R : $[\sqrt{3}, \sqrt{6}]$

(vi) D : $x \in (2n\pi, (2n+1)\pi) - \{2n\pi + \frac{\pi}{6}, 2n\pi + \frac{\pi}{2}, 2n\pi + \frac{5\pi}{6}, n \in I\}$ and Range is $(-\infty, \infty) - \{0\}$

(vii) D : $[-4, \infty) - \{5\}$; R : $\left(0, \frac{1}{6}\right) \cup \left(\frac{1}{6}, \frac{1}{3}\right]$ (viii) $\left[\frac{\pi}{4}, \pi\right)$ (ix) $\left[\frac{1}{2}, 1\right]$

46. **(**b) (i), (iii) are identical. **48.** $f(x) = \frac{x+1}{x-1}$ **49.** $\left(0, \frac{5}{3}\right)$ **50.** $[0, 4)$

51. $\begin{cases} -x \;; -2 \le x < 0 \\ 0 \;; 0 \le x < 1 \\ 2(x-1) \;; 1 \le x \le 2 \end{cases}$ **52.** $\left\{\frac{1}{4}\right\}$

53. A = $\left\{0, \log_3 7, \frac{129}{64}, \frac{33}{16}, \frac{9}{4}, 3, 5, 6\right\}$ and since $f(x)$ is not bijective therefore $f^{-1} : B \to A$ is not possible.

57. $f_o(x) \begin{cases} -x\tan x, & -\frac{\pi}{2} \le x \le 0 \\ -\frac{\pi}{2}[-x], & -2 \le x < -\frac{\pi}{2} \end{cases}$; $f_e(x) = \begin{cases} x\tan x, & -\frac{\pi}{2} \le x \le 0 \\ \frac{\pi}{2}[-x], & -2 \le x < -\frac{\pi}{2} \end{cases}$

58. $fog(x) = \begin{cases} -(1+x), & 1 \le x \le 0 \\ x-1, & 0 < x \le 2 \end{cases}$; $gof(x) = \begin{cases} x+1, & 0 \le x < 1 \\ 3-x, & 1 \le x \le 2 \\ x-1, & 2 < x \le 3 \\ 5-x, & 3 < x < 4 \end{cases}$; $fof(x) = \begin{cases} x, & 0 \le x \le 1 \\ 4-x, & 3 \le x < 4 \end{cases}$;

$gog(x) = \begin{cases} -x, & -1 \le x \le 0 \\ x, & 0 < x \le 2 \\ 4-x, & 2 < x \le 3 \end{cases}$

59. Domain : $[1, 2]$; Range : $[\ell n\, 2, \ell n\, 4)$

60. (i) neither even nor odd (ii) even (iii) even (iv) even (v) odd

61. (i) π (ii) 2 (iii) $\frac{2\pi}{3}$ (iv) 2π (v) $2^n \pi$ (vi) π **62.** $f(x) = 2x^2$ **63.** 6016

EXERCISE (Level-4)

SECTION-A

1. (D)	**2.** (A)	**3.** (D)	**4.** (C)	**5.** (C)	**6.** (B)	**7.** (C)
8. (A)	**9.** (D)	**10.** (C)	**11.** (C)	**12.** (B)	**13.** (B)	**14.** (C)
15. (D)	**16.** (D)	**17.** (C)	**18.** (C)	**19.** (B)	**20.** (B)	**21.** (B)
22. (C)	**23.** (D)	**24.** (C)	**25.** (C)	**26.** (B)	**27.** (A)	**28.** (D)
29. (Bonus)	**30.** (A)	**31.** (D)	**32.** (B)	**33.** (B)	**34.** (C)	**35.** (D)
36. (B)	**37.** (B)	**38.** (A)	**39.** (C)	**40.** (D)	**41.** (C)	**42.** (C)
43. (B)	**44.** (D)	**45.** (D)	**46.** (B)	**47.** 18.00	**48.** 1440	**49.** (C)
50. (D)	**51.** (A)	**52.** (B)	**53.** (B)			

SECTION-B

1. (A) **2.** (C) **3.** $\left[-\frac{\pi}{2},-\frac{\pi}{10}\right]\cup\left[\frac{3\pi}{10},\frac{\pi}{2}\right]$ **4.** A → P,R,S; B → Q,S; C → Q,S; D → P,R,S

5. (A) **6.** (B) **7.** (This question was awarded as bonus) **8.** (A,B,C) **9.** (A,B,C)

10. 119.00 **11.** (A) **12.** (C) **13.** 1.00 **14.** 19.00 **15.** (A,B)

16. (B,C,D) **17.** (C) **18.** 8 **19.** (C) **20.** (A, B)

EXERCISE (Level-5)

1. $a = 3$ **2.** $\alpha \in \phi$ **3.** $\{x \in R \mid x \notin [-1, 0)\}$ **4.** $\frac{\pm 3 \pm \sqrt{5}}{2}$ **5.** $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

6. 11 **7.** 20 **10.** No **11.** $R - (0, 1) \cup \{1, 2, 3, 4, 5, 6, 7\} \cup (7, 8)$

12. $P(x) = x + 3$, $Q(x) = -(x + 2)$ **13.** $(-4, -3) \cup (4, \infty)$ **14.** (B) **15.**(A)

16. n^n, $n!$