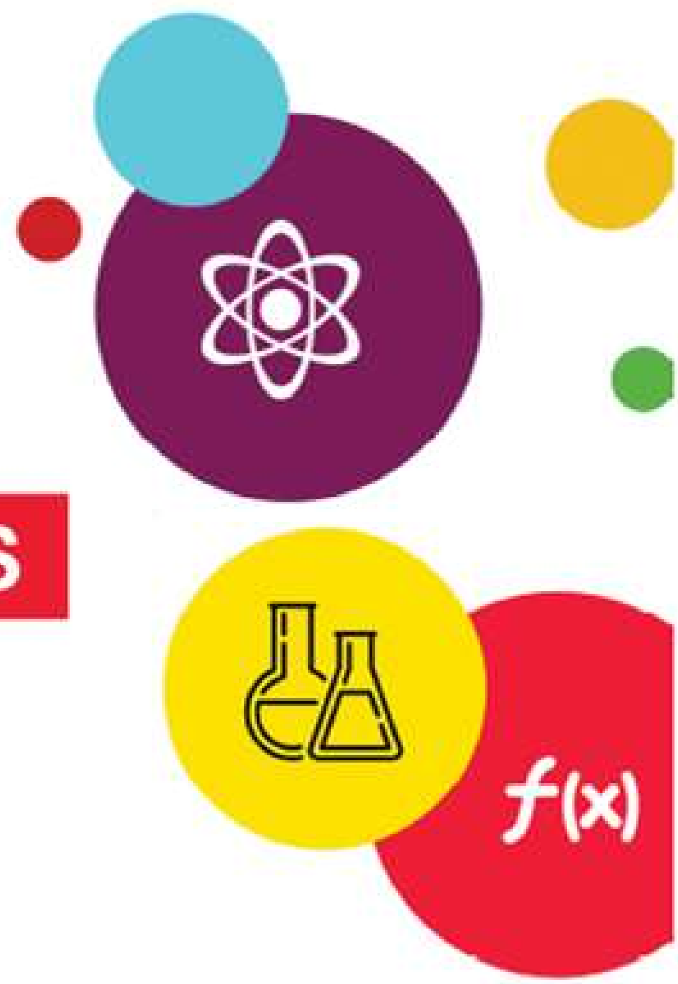
STUDY MATERIAL FOR

JEE Main

PHYSICS

CHEMISTRY

MATHEMATICS



CP PUBLICATION

CAREER POINT

Study Material for JEE Main preparation

Prepared by Career Point Kota Experts

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Function

This chapter covers the following syllabus

- Numbers and their sets
- Interval
- Function
- Classification of Functions
- Value of the Function
- Equal Function
- Kinds of functions
- Composite Functions
- Inverse Functions
- Domain & Range of Functions
- Functions & their graphs

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

- Write Question Number (QN) which you are unable to solve at your own in **column A**.
- After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.
- Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Exercise-1		
Exercise-2		
Exercise-3		
Exercise-4		
Exercise-5		

Revision Strategy:

Whenever you wish to revision this chapter, follow the following steps -

- **Step-1:** Review your theory notes.
- **Step-2:** Solve Questions of Column A
- **Step-3:** Solve Questions of Column B
- **Step-4:** Solve questions from other Question Bank, Problem book etc.

Function

KEY CONCEPT

1. Numbers and their Sets

(a) Natural Numbers : N = {1,2,3,4,...}

(b) Whole Numbers : W = {0,1,2,3,4,....}

(c) Integer Numbers :

I or Z = {...–3,–2,–1, 0,1,2,3,...},

Z^+ = {1,2,3,....}, Z^- = {–1,–2,–3,....}

Z_0 = {± 1, ± 2, ± 3,....}

(d) Rational Numbers :

$Q = \{\frac{p}{q};\ p, q \in z, q \neq 0\}$

Ex. $\{1, \frac{5}{3}, -10, 10^5, \frac{22}{7}, \frac{-20}{3}, 0 \ldots\}$

Note :

- In rational numbers the digits are repeated after decimal.
- 0 (zero) is a rational number.

(e) Irrational numbers: The numbers which are not rational or which can not be written in the form of p/q ,called irrational numbers

Ex. $\{\sqrt{2}, \sqrt{3}, 2^{1/3}, 5^{1/4}, \pi, e, \ldots\}$

Note:

- In irrational numbers, digits are not repeated after decimal.
- π and e are called special irrational quantities.
- ∞ is neither a rational number nor a irrational number.

(f) Real Numbers : {x, where x is rational and irrational number}

Ex. $R = \{1, 1000, 20/6, \pi, \sqrt{2}, -10, -\frac{20}{3}, \ldots\}$

(g) Positive Real Numbers: $R^+ = (0,\infty)$

(h) Negative Real Numbers : $R^- = (-\infty,0)$

(i) R_0 : all real numbers except 0 (Zero).

(j) Imaginary Numbers : C = {i,ω,....}

(k) Prime Numbers :

These are the natural numbers greater than 1 which is divisible by 1 and itself only, called prime numbers.

2,3,5,7,11,13,17,19,23,29,31,37,41,...

(l) Even Numbers : E = {0,2,4,6,....}

(m) Odd Numbers : O = {1,3,5,7,....}

2. Interval

The set of the numbers between any two real numbers is called **interval**.

(a) Close Interval :

$[a, b] = \{x, a \le x \le b\}$

(b) Open Interval:

(a, b) or $]a, b[= \{x, a < x < b\}$

(c) Semi open or semi close interval:

$[a, b[$ or $[a, b) = \{x; a \le x < b\}$

$]a, b]$ or $(a, b] = \{x; a < x \le b\}$

3. Function

Let A and B be two given sets and if each element $a \in A$ is associated with a unique element $b \in B$ under a rule f, then this relation is called **function**.

Here b, is called the image of a and a is called the pre- image of b under f.

Note :

- Every element of A should be associated with B but vice-versa is not essential.
- Every element of A should be associated with a unique (one and only one) element of but any element of B can have two or more relations in A.

3.1 Representation of Function :

It can be done by three methods :

(A) By Mapping

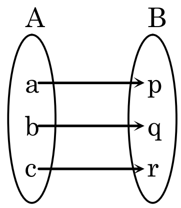
(B) By Algebraic Method

(C) In the form of Ordered pairs

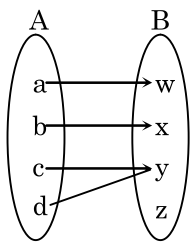
(A) Mapping :

It shows the graphical aspect of the relation of the elements of A with the elements of B.

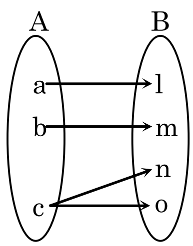
Ex. f_1**:**



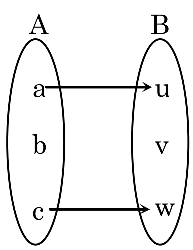
f_2 **:**



f_3 **:**



f_4 **:**



In the above given mappings rule f_1 and f_2 shows a function because each element of A is associated with a unique element of B. Whereas f_3 and f_4 are not function because in f_3, element c is associated with two elements of B, and in f_4 , b is not associated with any element of B, which do not follow the definition of function. In f_2, c and d are associated with same element, still it obeys the rule of definition of function because it does not tell that every element of A should be associated with different elements of B.

(B) Algebraic Method :

It shows the relation between the elements of two sets in the form of two variables x and y where x is independent variable and y is dependent variable.

If A and B be two given sets

$A = \{1, 2, 3\}$, $B = \{5, 7, 9\}$

then $f : A \rightarrow B$, $y = f(x) = 2x + 3$.

(C) In the form of ordered pairs :

A function $f : A \rightarrow B$ can be expressed as a set of ordered pairs in which first element of every ordered pair is a member of A and second element is the member of B. So f is a set of ordered pairs (a, b) such that :

(i) a is an element of A

(ii) b is an element of B

(iii) Two ordered pairs should not have the same first element.

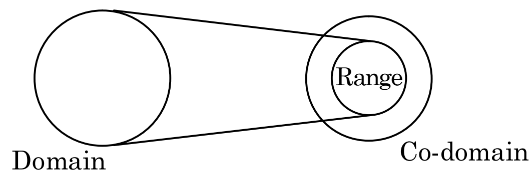
3.2 Domain, Co-domain and Range :

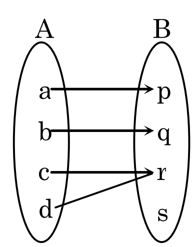
If a function f is defined from a set of A to set B then for f: $A \rightarrow B$ set A is called the **domain** of function f and set B is called the **co-domain** of function f. The set of the f- images of the elements of A is called the **range** of function f.

In other words, we can say

Domain = All possible values of x for which f(x) exists.

Range = For all values of x, all possible values of f(x).





Domain = {a, b, c, d} = A

Co-domain = {p, q, r, s} = B

Range = {p, q, r}

3.3 Algebra of functions :

Let f and g be two given functions and their domain are D_f and D_g respectively, then the sum, difference, product and quotient functions are defined as :

(a) $(f + g)(x) = f(x) + g(x),\ \forall\ x \in D_f \cap D_g$

(b) $(f - g)(x) = f(x) - g(x),\ \forall x \in D_f \cap D_g$

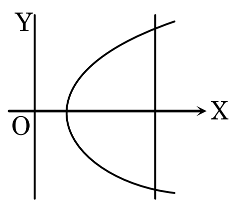
(c) $(f \,.\, g)(x) = f(x) \,.\, g(x),\ \forall\ x \in D_f \cap D_g$

(d) $(f/g)(x) = \dfrac{f(x)}{g(x)}$; $g(x) \neq 0,\ \forall\ x \in D_f \cap D_g$

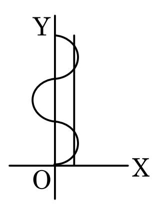
3.4 Testing for a function : A relation f : A B is a function or not, it can be checked by following methods.

(a) See Article 3 (a) & 3 (b)

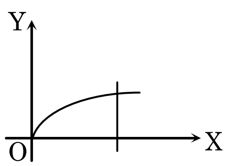
(b) Vertical Line Test : If we are given a graph of the relation then we can check whether the given relation is function or not . If it is possible to draw a vertical line which cuts the given curve at more than one point then given relation is not a function and when this vertical line means line parallel to Y – axis cuts the curve at only one point then it is a function.



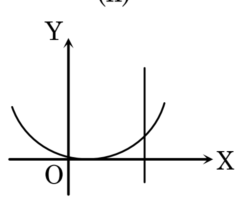
(i)



(ii)



(iii)



(iv)

fig. (iii) and (iv) represents a function.

4. Classification of Function

4.1 Algebric and Transcendental function :

4.1.1 Algebric Function :

The function which consists of sum, difference, product, quotient, power or roots of a variable is called the algebric function.

Algebric functions further can be classified as –

(a) Polynomial or integral Function :

The function of the form

$f(x) = a_0 x^n + a_1 x^{n-1} + ...+ a_{n-1}x + a_n,\ a_0 \neq 0$

Where $a_0, a_1, a_2,...a_n$ are constants and $n \in N$ is called polynomial or Integral Functions.

(i) $f(x) = C$, is a polynomial of zero power or a constant function.

(ii) $f(x) = a\ x + b$, is a polynomial of power one or a linear function.

(iii) $f(x) = ax^2 + bx + c$, is a polynomial of two power or a quadratic function and so on.

(b) Rational Function :

The quotient of two polynomial functions is called the Rational function.

(c) Irrational Function:

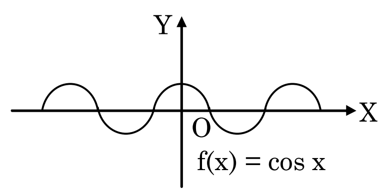
A function which is not rational is called Irrational Function.

4.1.2 Transcendental Function: The function which is not algebric is called transcendental function.

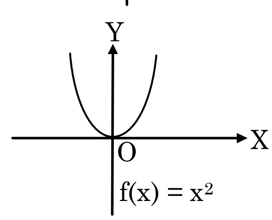
4.2 Even or Odd Function:

4.2.1 Even function :

If we put (–x) in place of x in the given function and if $f(-x) = f(x)$, $x \in$ domain then function f(x) is called even function.



$f(x) = \cos x$



$f(x) = x^2$

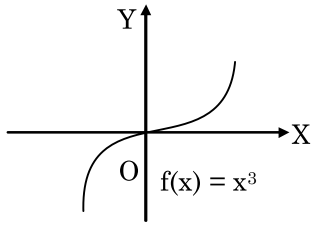
Note :

➢ The graph of even function is always symmetric with respect to y-axis.

4.2.2 Odd function :

If we put (–x) in place of x in the given function and if $f(-x) = -f(x),\ \forall\ x \in$ domain then f (x) is called odd function.

Then f(x) is called odd function.



Note :

- The graph of odd function is always symmetric with respect to origin.

Properties of Even and Odd Function :

(a) The product of two even functions is even function.

(b) The sum and difference of two even functions is even function.

(c) The sum and difference of two odd functions is odd function.

(d) The product of two odd functions is even function.

(e) The product of an even and an odd function is odd function.

(f) The sum of even and odd function is neither even nor odd function.

(g) It is not essential that every function is even or odd. It is possible to have some functions which are neither even nor odd function.

Note :

- Zero function $f(x) = 0$ is the only function which is even and odd both.

4.3 Explicit and Implicit function :

(a) Explicit Function :

A function is said to be explicit if it can be expressed directly in terms of the independent variable.

$y = f(x)$ or $x = \phi(y)$

eg. $f(x) = x^2 \sin x$

(b) Implicit Function :

A function is said to be implicit if it can not be expressed directly in terms of the independent variable.

eg. $x^3 + y^3 + 3axy + c = 0$, $x^y + y^x = a^b$

4.4 Continuous and Discontinuous Function :

(a) Continuous Function :

A function is said to be continuous function in an interval I if we are not required to lift the pen or pencil off the paper i.e. there is no gap or break or jump in the graph.

(b) Discontinuous Function :

A function is said to be discontinuous if there is a break or gap or jump in the graph of the function at any point.

4.5 Increasing and Decreasing Function :

(a) Increasing Function :

A function f(x) is called increasing function in the domain D if the value of the function does not decrease by increasing the value of x.

so $x_1 > x_2 \Rightarrow f(x_1) \geq f(x_2)\ \forall\ x_1, x_2 \in$ domain

or $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)\ \forall\ x_1, x_2 \in$ domain

A function is called strictly increasing if

if $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$

or $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)\ \forall\ x_1, x_2 \in$ domain

(b) Decreasing Function :

A function f (x) is said to be decreasing function in the domain D if the value of the function does not increase by increasing the value of x (variable).

so if $x_1 > x_2 \Rightarrow f(x_1) \leq f(x_2)$

or $x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)\ \forall\ x_1, x_2 \in D$

A function is called strictly decreasing if

if $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

or $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)\ \forall\ x_1, x_2 \in D$

Note :

- It is not essential for any function to be increasing or decreasing. There are some functions which are neither increasing nor decreasing i.e. function is increasing in one part of given interval and decreasing in second part.

4.6 Greatest Integer Function :

A function is said to be greatest integer function if it is of the form of $f(x) = [x]$ where $[x]$ = integer equal or less than x.

$f(x) = y = [x]$

$0 \leq x < 1 \Rightarrow y = 0$

$1 \leq x < 2 \Rightarrow y = 1$

$2 \leq x < 3 \Rightarrow y = 2$

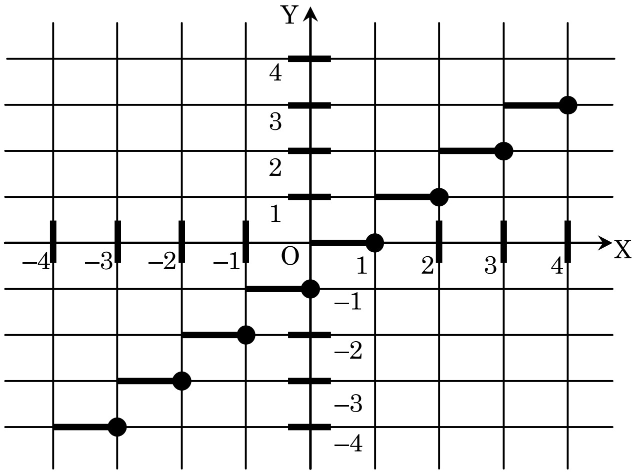
$\vdots$

and so on

Note : Important Identities :

- $[x] \leq x$ **(**This is always true**)**

- $[x] + 1 > x$



4.7 Periodic Function:

A function is said to be periodic function if its each value is repeated after a definite interval.

So a function f(x) will be periodic if a positive real number T exist such that,

$f(x + T) = f(x), \forall x \in$ Domain

Here the least positive value of T is called the period of the function. Clearly

$f(x) = f(x + T) = f(x + 2T) = f(x + 3T) = ...$

For example, sin x, cos x, tan x are periodic functions with period 2π, 2π & π respectively.

Note:

- If function f (x) has period T then

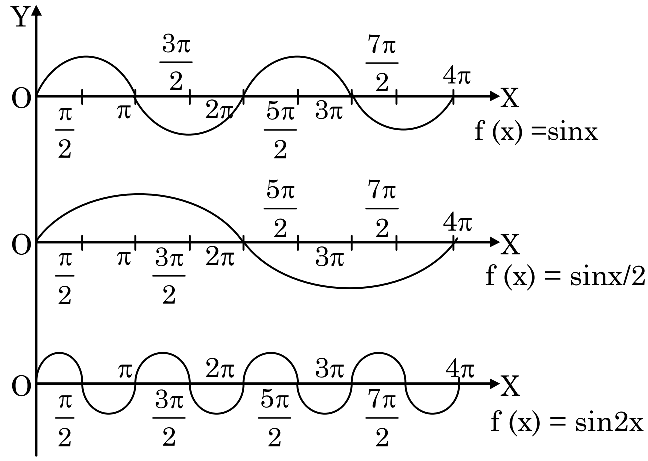
  f (nx) has period $\frac{T}{n}$

  f(x/n) has period nT.

  f (ax + b) has period $\frac{T}{|a|}$

- If the period of f (x) and g (x) are same (T) then the period of a f (x) + bg (x) will also be T.
- If the period of f(x) is T_1 and g (x) has T_2, then the period of $f(x) \pm g(x)$ will be LCM of T_1 and T_2 provided it satisfies the definition of periodic function.

  The graphs of f(x) = sin x, f(x) = sin x/2, and f(x) = sin 2x are being compared to find the period.



5. Value of the Function

If y = f(x) is any function defined in R, then for any given value of x (say x = a), the value of the function f(x) can be obtained by substituting x = a in it and it is denoted by f(a).

6. Equal Function

Two functions $f : A \to B$ and $g : C \to D$ are called equal functions if and only if

(a) domain of f = domain of g

(b) co-domain of f = co-domain of g

(c) $f(x) = g(x)$, $\forall x \in$ domain

7. Kinds of Functions

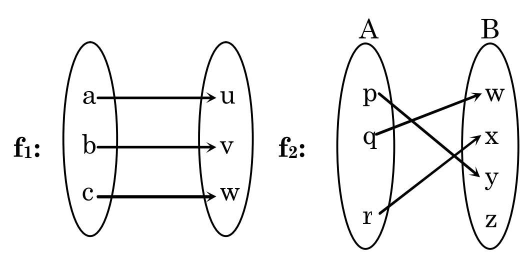
7.1 One - One function or Injection :

A function $f : A \to B$ is said to be one- one if different elements of A have different images in B. Therefore for any two elements x_1, x_2 of a set A,

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

or $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

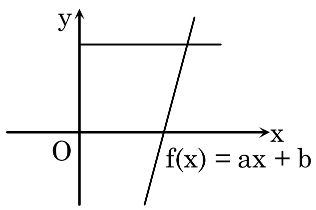
then function is one-one.

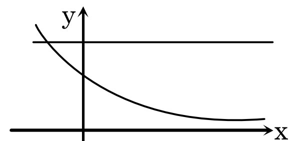


The above given diagrams f_1 & f_2 shows one-one function.

Note :

- If function is given in the form of ordered pairs and if no two ordered pairs have same second element then function is one-one.
- If the graph of the function $y = f(x)$ is given, and each line parallel to x-axis cuts the given curve at maximum one point then function is one-one.





$f(x) = a^x$ $(0 < a < 1)$

Examples of One-One Function –

(i) $f : R \to R$, $f(x) = x$,

(ii) $f : R \to R$, $f(x) = ax + b$

(iii) $f : R \to R$, $f(x) = ax^n + b$, n is odd positive integer

(iv) $f : R \to R$, $f(x) = x \mid x \mid$

(v) $f : R \to R$, $f(x) = e^x$,

(vi) $f : R \to R$, $f(x) = a^x$ $(a > 0)$

(vii) $f : R \to R$, $f(x) = \sinh x$,

(viii) $f : R \to R$, $f(x) = \tanh (x)$

(ix) $f : R_0 \to R$, $f(x) = 1/x$,

(x) $f : R^+ \to R$, $f(x) = \log x$,

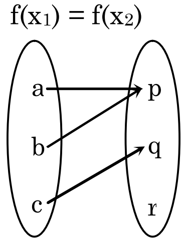
(xi) $f : R_0 \to R$, $f(x) = \log_a x$ $(a>0)$

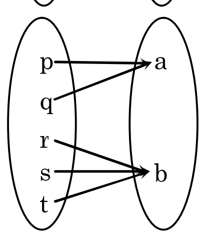
7.2 Many-one Function :

A function $f : A \to B$ is called many- one, if two or more different elements of A have the same f-image in B.

therefore f : A B is many-one if

$x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$

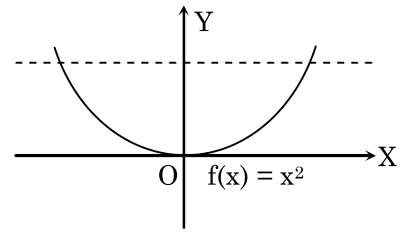


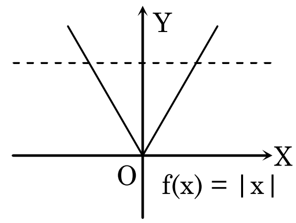


The above given arrow-diagrams show many-one function.

(a) If function is given in the form of set of ordered pairs and the second element of atleast two ordered pairs are same then function is many- one.

(b) If the graph of $y = f(x)$ is given and the line parallel to x-axis cuts the curve at more than one point then function is many-one.





Example of many-one function :

(i) $f : R \to R$, $f(x) = C$, where C is a constant

(ii) $f : R \to R$, $f(x) = x^2$

(iii) $f : R \to R$, $f(x) = ax^2 + b$,

(iv) $f : R \to R$, $f(x) = |x|$,

(v) $f : R \to R$, $f(x) = x + |x|$

(vi) $f : R \to R$, $f(x) = x - |x|$

(vii) $f : R \to R$, $f(x) = \cosh x$

(viii) $f : R \to R$, $f(x) = [x]$,

(ix) $f : R \to R$, $f(x) = x - [x]$

Where [x] is greatest integer function.

Methods to check trigonometrical functions to be one-one or many-one.

(a) If the domain of the function is in one quadrant then trigonometrical functions are always one-one.

(b) If trigonometrical function changes its sign in two consecutive quadrants then it is one-one but if it does not change the sign then it is many one.

$f : (0, \pi)$, $f(x) = \sin x$ many-one

and $f : (0, \pi)$, $f(x) = \cos x$ one-one

(c) In three consecutive quadrants trigonometrical functions are always one-one.

7.3 Onto function or Surjection :

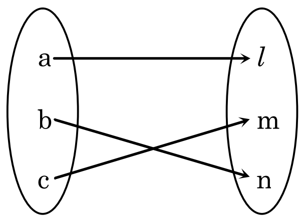
A function $f : A \to B$ is onto if the each element of B has its pre- image in A. Therefore if

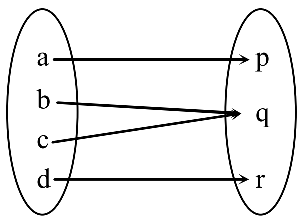
$f^{-1}(y) \in A, \forall y \in B$ then function is onto.

In other words.

Range of f = Co-domain of f.

The following arrow-diagram shows onto function.





Examples of onto function :

(i) $f : R \to R$, $f(x) = x$,

(ii) $f : R \to R$, $f(x) = ax + b$, $a \neq 0$, $b \in R$

(iii) $f : R \to R$, $f(x) = x^3$

(iv) $f : R \to R$, $f(x) = x|x|$

(v) $f : R \to R^+$, $f(x) = e^x$

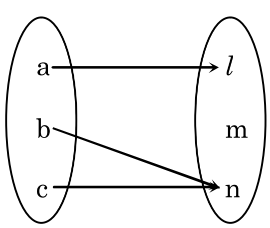
(vi) $f : R^+ \to R$, $f(x) = \log x$.

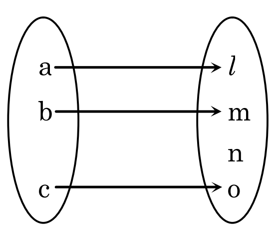
7.4 Into function :

A function $f : A \to B$ is into if there exist atleast one element in B which is not the f-image of any element in A. Therefore, atleast one element of B such that $f^{-1}(y) = \phi$ then function is into. In other words

Range of f $\neq$ co-domain of f

The following arrow-diagram shows into function.





Examples of into function :

(i) $f : R \to R$, $f(x) = x^2$

(ii) $f : R \to R$, $f(x) = |x|$

(iii) $f : R \to R$, $f(x) = c$ (c is constant)

(iv) $f : R \to R$, $f(x) = \sin x$

(v) $f : R \to R$, $f(x) = \cos x$

(vi) $f : R \to R$, $f(x) = e^x$

(vii) $f : R \to R$, $f(x) = a^x$, $a > 0$

Note :

➢ For a function to be onto or into depends mainly on their co-domain.

Now, we can classify the function further into four categories :

7.5 One-one onto function or bijection :

A function f is said to be one-one onto if f is one-one and onto both.

7.6 One-one into function :

A function is said to be one- one into if f is one-one but not onto.

7.7 Many one-onto function :

A function $f : A \to B$ is many one-onto if f is onto but not one one.

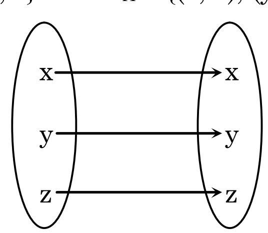
7.8 Many one-into function :

A function is said to be many one-into if it is neither one-one nor onto.

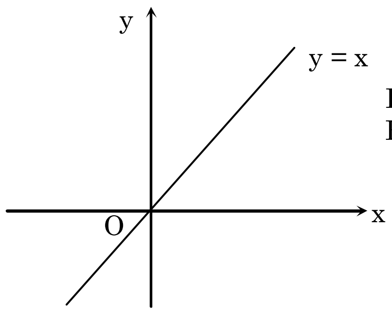
7.9 Identity Function :

Let A be any set and the function $f : A \to A$ be defined as $f(x) = x$, $\forall x \in A$ i.e. if each element of A is mapped by itself then f is called the identity function. It is represented by I_A.

If $A = \{x, y, z\}$ then $I_A = \{(x, x), (y, y), (z, z)\}$



Graph -

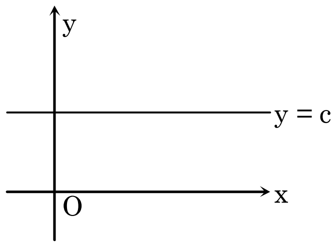


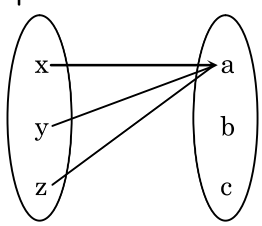
Domain = R
Range = R

7.10 Constant function :

If in a function $f : A \to B$ $f(a) = c, \forall a \in A$. then it is a constant function.

$f(x) = c, \forall x \in R$



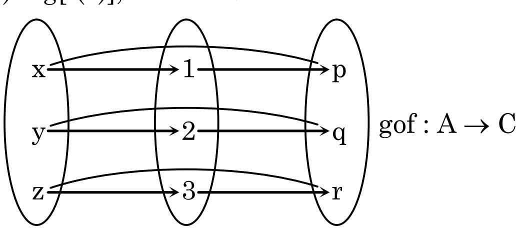


The range of constant function contains only one element.

8. Composite Function

If $f : A\to B$ and $g : B\to C$ are two function then the composite function of f and g , $gof : A \to C$ will be defined as

$gof(x) = g[f(x)], \forall x \in A$.



Note :

- Function gof will exist only when range of f is the subset of domain of g.
- gof (x) is simply the g-image of f(x), where f(x) is f-image of elements $x \in A$.
- fog does not exist here because range of g is not a subset of domain of f.

Properties of composite function :

(a) If f and g are two functions then for composite of two functions $fog \neq gof$.

(b) Composite functions obeys the property of associativity i.e. fo (goh) = (fog) oh.

(c) Composite function of two one-one onto functions if exist, will also be a one-one onto function.

9. Inverse Function

If $f : A \to B$ be a one-one onto (bijection) function, then the mapping $f^{-1} : B\to A$ which associates each element $b \in B$ with element $a \forall A$, such that $f(a) = b$, is called the **inverse function** of the function $f : A \to B$

$f^{-1} : B \to A, \; f^{-1}(b) = a \Rightarrow f(a) = b$

In terms of ordered pairs inverse function is defined as -

$f^{-1} = \{(b, a) \mid (a, b) \in f\}$

Note :

- For the existence of inverse function, it should be one-one and onto.

Properties :

(a) Inverse of a bijection is also a bijection function.

(b) Inverse of a bijection is unique.

(c) $(f^{-1})^{-1} = f$

(d) If f and g are two bijections such that (gof) exists then $(gof)^{-1} = f^{-1}og^{-1}$

(e) If $f : A\to B$ is a bijection then $f^{-1}: B\to A$ is an inverse function of f .

$f^{-1}of = I_A$ and $fof^{-1} = I_B$.

Here I_A, is an identity function on set A, and I_B, is an identity function on set B.

10. Domain & Range of Some Standard Function

Function	Domain	Range
Polynomial function	R	R
Identity function x	R	R
Constant function c	R	{c}
Reciprocal f^n 1/x	R_0	R_0
Signum function	R	{–1,0,1}
ax + b ; a, b $\in$ R	R	R
$ax^3 + b$; a, b $\in$ R	R	R
x^2, |x|	R	$R^+ \cup \{0\}$
x^3, x |x|	R	R
x + |x|	R	$R^+ \cup \{0\}$
x – |x|	R	$R^- \cup \{0\}$
[x]	R	Z
x – [x]	R	[0, 1)
|x|/ x	R_0	{–1, 1}
$\sqrt{x}$	$[0, \infty)$	$[0, \infty)$
a^x	R	R^+
log x	R^+	R
sin x	R	[–1, 1]
cos x	R	[–1, 1]
tan x	$R - \{(2n+1)\pi/2 \mid n \in Z\}$	R
cot x	$R - \{n\pi \mid n \in Z\}$	R
sec x	$R - \{(2n+1)\pi/2 \mid n \in Z\}$	R – (–1,1)
cosec x	$R - \{n\pi \mid n \in Z\}$	R – (–1,1)
$\sin^{-1} x$	[–1, 1]	$[-\pi/2, \pi/2]$
$\cos^{-1} x$	[–1, 1]	$[0, \pi]$
$\tan^{-1} x$	R	$(-\pi/2, +\pi/2)$
$\cot^{-1} x$	R	$(0, \pi)$
$\sec^{-1} x$	R – (–1,1)	$[0, \pi] - \{\pi/2\}$
$\mathrm{cosec}^{-1} x$	R – (–1,1)	$(-\pi/2, \pi/2] - \{0\}$

11. Some Functions & Their Graphs

11.1 Classification of function :

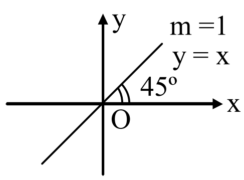
- Function
  - Algebric
    - Polynomial
    - Rational
    - Irrational
    - Piecewise
      - Modulus
      - Greatest integer function
      - Fractional part
      - Signum
  - Transcendental
    - Trigonometric
    - Exponential
    - Logarithmic
    - Inverse Trigonometric

Graphs of Function :

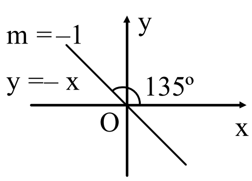
(1) Polynomials

(A) $y = f(x) = mx$, where m is a constant ($m \neq 0$)

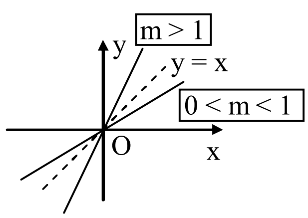
(i)



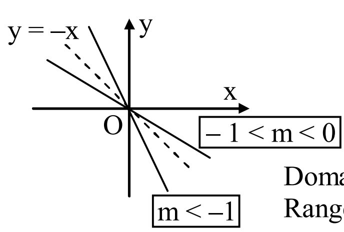
(ii)



(iii)



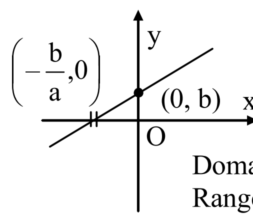
(iv)



Domain = R
Range = R

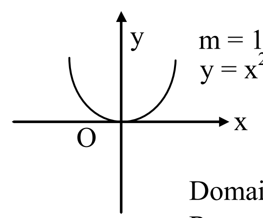
$y = mx$ is continuous function & one-one function ($m \neq 0$)

(v) $y = ax + b$, where a & b are constant

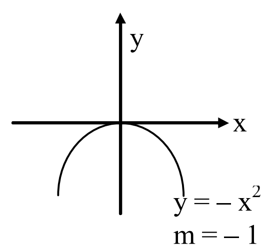


Domain = R
Range = R

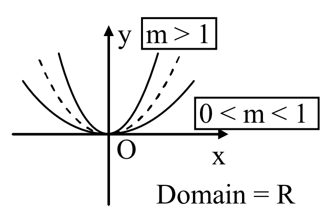
(B) $y = mx^2$, ($m \neq 0$), $x \in R$



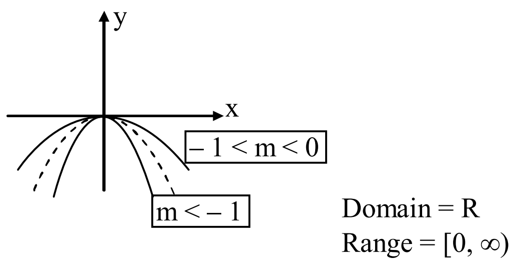
Domain = R
Range = $[0, \infty]$



Domain = R
Range = $(\infty, 0]$



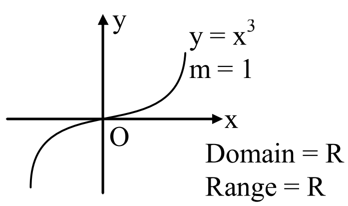
Domain = R
Range = $[0, \infty)$

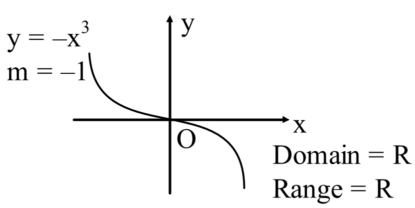


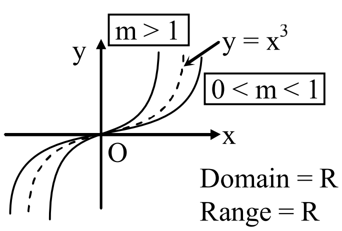
$y = mx^2$ is continuous, even function and many-one function.

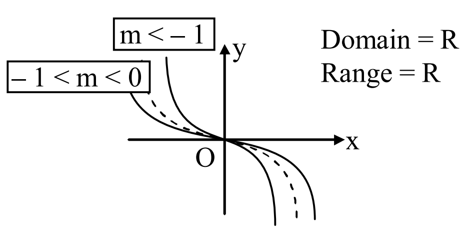
Graph is symmetric about y-axis

(C) $y = mx^3$, $(m \neq 0)$, $x \in R$





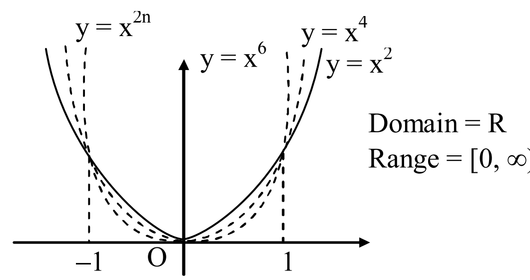




$y = mx^3$ is continuous, odd one-one & increasing function.

Graph is symmetric about origin.

(D) $y = x^{2n}$, $n \in N$



$f(x) = x^{2n}$ is an even, continuous function. It is many one function.

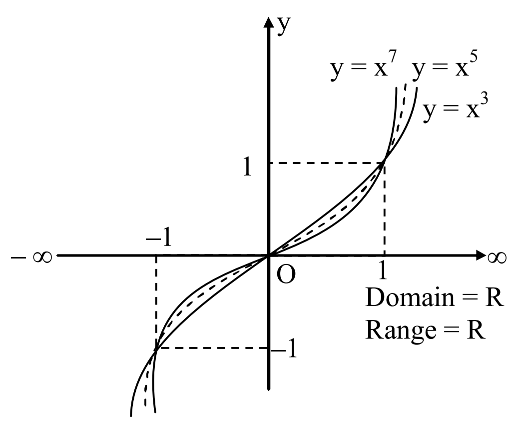
From the graph :

For $|x| \leq 1 \Rightarrow x^6 \leq x^4 \leq x^2$

For $|x| > 1 \Rightarrow x^6 > x^4 > x^2$

Graph is symmetric about y-axis

(E) $y = x^{2n+1}$, $n \in N$



$f(x) = x^{2n+1}$, $n \in N$ is an odd continuous function. It is always one-one function.

From the graph :

For $x \in (-\infty, -1] \cup [0, 1]$

$\Rightarrow x^3 \geq x^5 \geq x^7$

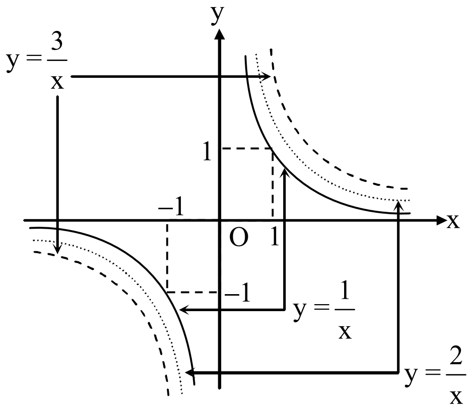
For $x \in (-1, 0) \cup (1, \infty)$

$\Rightarrow x^3 < x^5 < x^7$

Graph is symmetric about origin.

(2) Rational Function :

(A) $y = \frac{m}{x}$, $x \neq 0$, $m \neq 0$



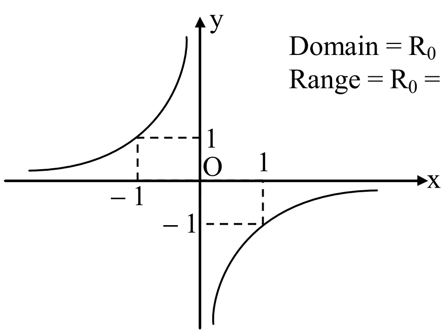
$y = \frac{m}{x}$ is odd function discontinuous at x = 0.

Domain = $R_0 = R - \{0\}$

Range = $R_0 = R - \{0\}$

This type of graph xy = m is called rectangular hyperbola.

(B) $y = -\frac{1}{x}$, $x \neq 0$



Domain = $R_0 = R - \{0\}$
Range = $R_0 = R - \{0\}$

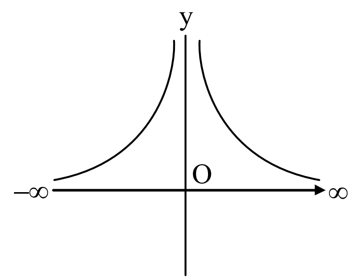
$y = -\frac{1}{x}$ is an odd & one-one function discontinuous at x = 0

(C) $y = \frac{1}{x^2}$, $x \neq 0$

$y = \frac{1}{x^2}$ is even function, many one and discontinuous at x = 0

Domain = R_0

Range = $R^+ = (0, \infty)$



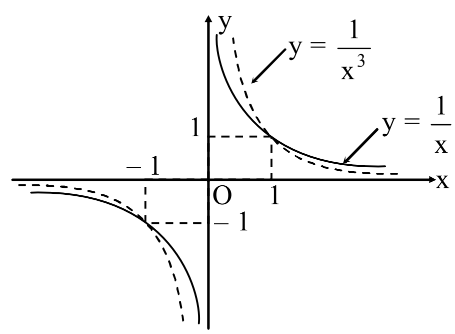
(D) $y = \frac{1}{x^{2n-1}}$, $n \in N$

An odd one-one function discontinuous at x = 0

From graph

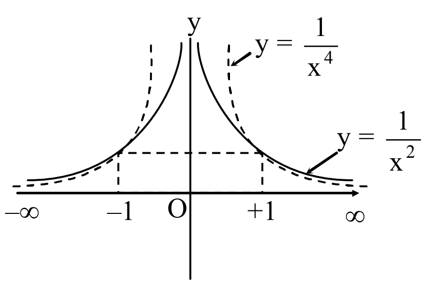
$0 < x < 1 \Rightarrow \frac{1}{x^3} > \frac{1}{x}$

$x > 1 \Rightarrow \frac{1}{x^3} < \frac{1}{x}$.



(E) $y = \frac{1}{x^{2n}}$, $n \in N$

$y = \frac{1}{x^{2n}}$ is an even function & many one, discontinuous at x = 0.



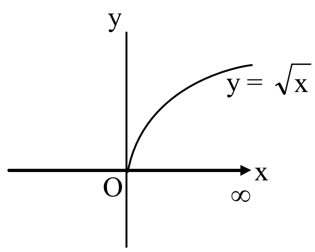
Domain = R_0
Range = R^+

(3) Irrational function

(A) $y = \sqrt{x} = x^{1/2}$

This function is neither even nor odd, one-one and continuous function in it's domain.

It is an increasing function.

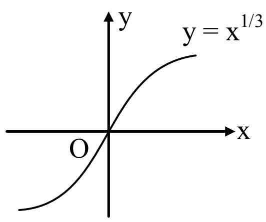


Domain = $[0, \infty)$

Range = $[0, \infty)$

(B) $y = \sqrt[3]{x} = x^{1/3}$

This function is an odd, continuous and one-one function. It is an increasing function.



Domain = R
Range = R

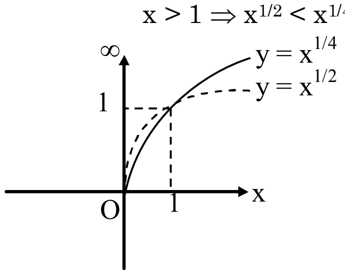
(C) $y = x^{1/2n}$, $n \in N$

This function is neither even nor odd, continuous and increasing function it is also one-one function

From graph

$0 < x < 1 \Rightarrow > \sqrt[2]{x} > \sqrt[4]{x}$

$x > 1 \Rightarrow x^{1/2} < x^{1/4}$



Domain = $R^+ = [0, \infty)$
Range = $R^+ = [0, \infty)$

(D) $y = x^{\frac{1}{2n+1}}$, $n \in N$

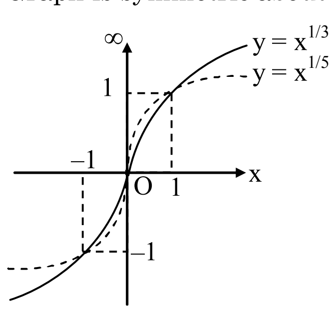
This function is an odd, continuous, one-one and increasing function

From graph

$x \in (-\infty, -1) \cup (0, 1)$, $x^{1/3} > x^{1/5}$

$x \in (-1, 0) \cup (1, \infty)$, $x^{1/3} < x^{1/5}$

Graph is symmetric about origin.



Domain = R
Range = R

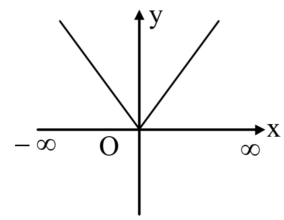
(4) Piecewise function (Special Function)

(A) Modulus function

$$y = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

This is an even, continuous, many one function.

Graph is symmetric about y-axis

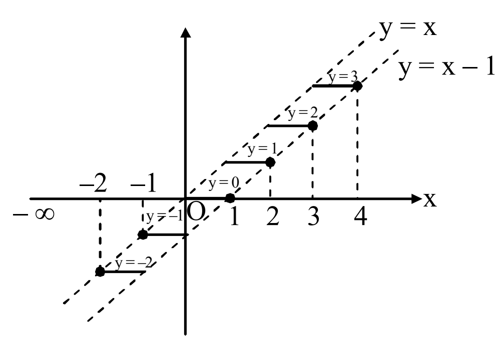


Domain = R

Range = $[0, \infty)$

(B) Greatest integer Function

[x] indicates the integral part of x which is nearest and smaller to x. It is also called step function.



$$y = [x] = \begin{cases} -1, & -1 \leq x < 0 \\ 0, & 0 \leq x < 1 \\ +1, & 1 \leq x < 2 \\ +2, & 2 \leq x < 3 \\ +3, & 3 \leq x < 4 \end{cases}$$

$\Rightarrow y = [x] = k, k \leq x < k + 1, k \in I$

Important Result from graph

$x - 1 < [x] \leq x$

(C) Fractional part of x

$y = \{x\}$

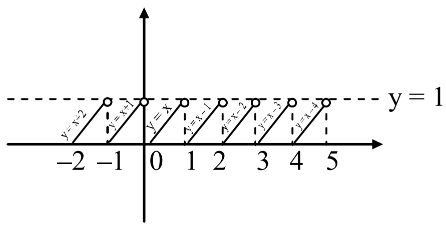
Any number is sum of integral & fractional part i.e. $x = [x] + \{x\}$

$\therefore$ $y = \{x\} = x - [x] = x - k, k \leq x < k + 1$

$$y = \{x\} = \begin{cases} x+1, & \ldots\ldots. \\ x, & 0 \le x < 1 \\ x-1, & 1 \le x < 2 \\ x-2, & 2 \le x < 3 \\ \ldots\ldots., & \ldots\ldots. \end{cases}$$

Domain = R

Range = [0, 1)

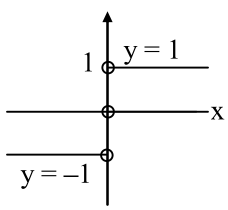


From graph $0 \le \{x\} < 1$

This function is a periodic function with period 1. This is also many-one function discontinuous at $x \in I$.

(D) $y = \text{sgn}(x) = \begin{cases} \dfrac{|x|}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases} = \begin{cases} x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}$

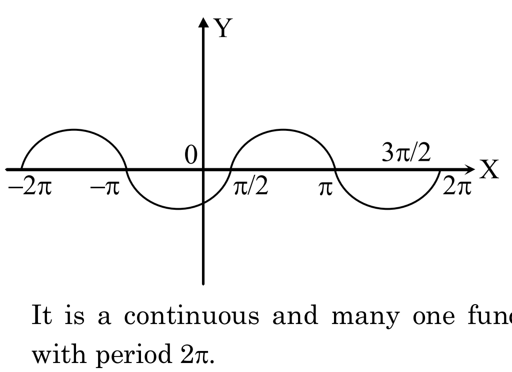
This is odd, many-one function discontinuous at x = 0



Domain = R
Range = {–1, 0, 1}

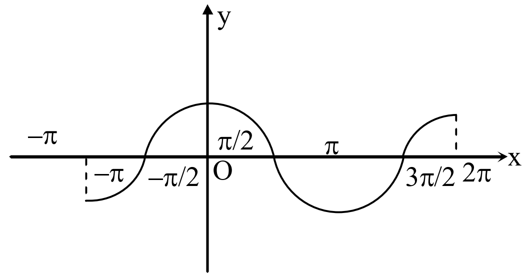
(5) Trigonometrical function :

(i) f (x) = sin x.



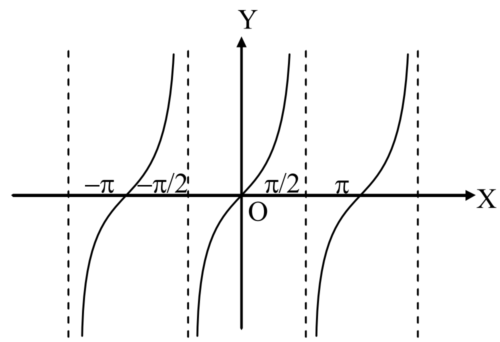
It is a continuous and many one function with period 2π.

(ii) f(x) = cos x



It is a continuous and many-one function having a period $2.\pi$

(iii) f(x) = tan x



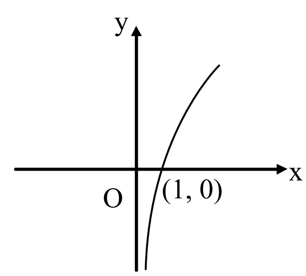
Domain = R – (2n+1) $\pi/2$, $n \in I$

Range = R

It is a discontinuous function at x = $(2n+1)\pi/2$, $n \in I$ and periodic with period π.

(6) Logarithmic function :

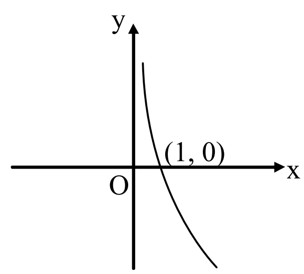
(i) f (x) = $\log_a x$ (a > 1)



Domain = R^+
Range = R

It is a continuous and one-one function.

(ii) f (x) = $\log_a x$ (a<1)

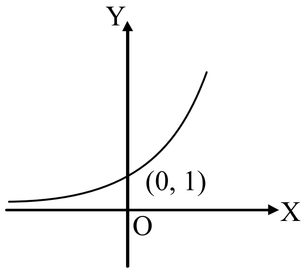


Domain = R^+
Range = R

It is a continuous and one-one function.

(7) Exponential function :

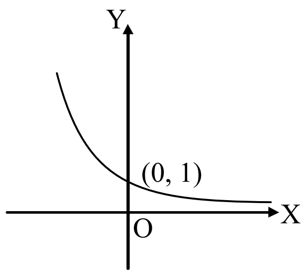
(i) $f(x) = a^x$ (a>1)



Domain = R
Range = R^+

It is a continuous and one-one function.

(ii) $f(x) = a^x$ (a<1)

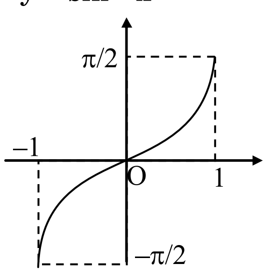


Domain = R
Range = R^+

It is a continuous and one-one function.

(8) Inverse Trigonometric Function :

$y = \sin^{-1} x$

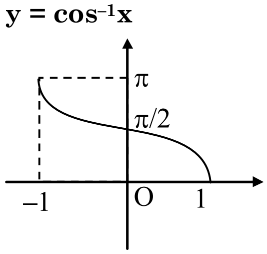


Domain = [–1, 1]

Range $= \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

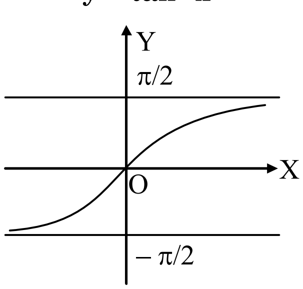
Odd function

$y = \cos^{-1} x$



Domain = [–1, 1]
Range = [0, π]
Neither even nor odd

$y = \tan^{-1} x$

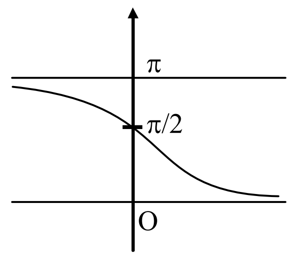


Domain = R

Range $= \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

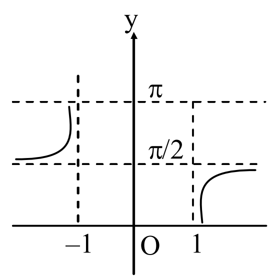
Odd function

$y = \cot^{-1} x$



Domain = R
Range = (0, π)
Neither even nor odd

$y = \sec^{-1} x$

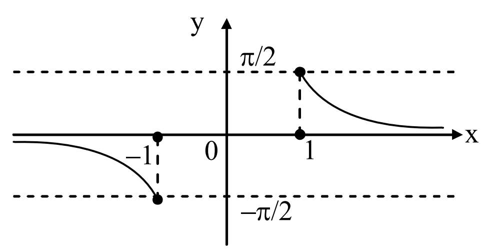


Domain = (–∞, –1] ∪ [1, ∞)

Range $= [0, \pi] - \left\{\frac{\pi}{2}\right\}$

Neither even nor odd

$y = \mathrm{cosec}^{-1} x$



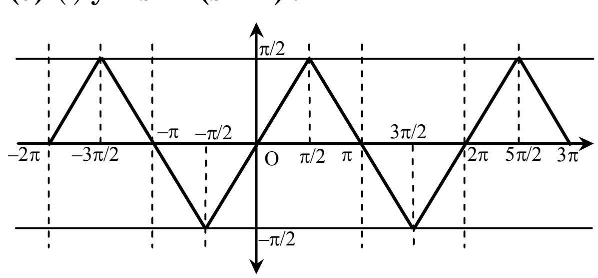
Domain = (–∞, –1] ∪ [1, ∞)

Range $= \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

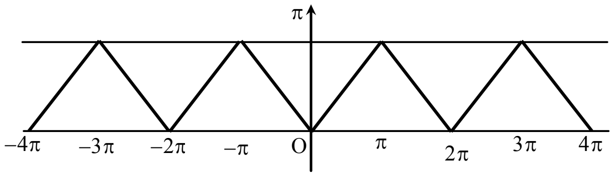
Odd function

All inverse trigonometric functions are monotonic.

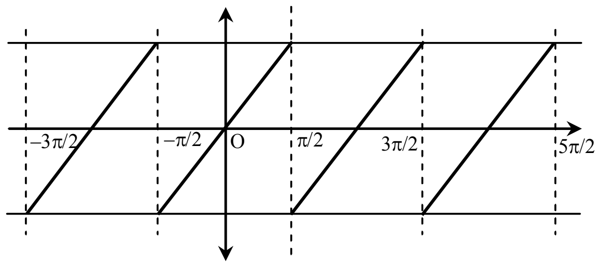
(9) (i) **$y = \sin^{-1}(\sin x)$:**



(ii) $y = \cos^{-1}(\cos x)$:



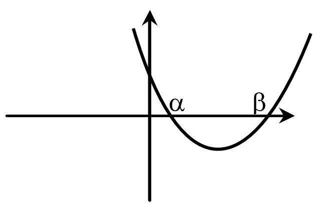
(iii) $y = \tan^{-1}(\tan x)$



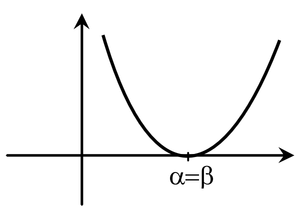
(10) Graph of Quadratic function

$f(x) = ax^2 + bx + c$, $D = b^2 - 4ac$

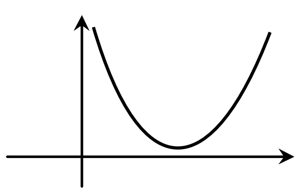
(i) $a > 0$; $D > 0$



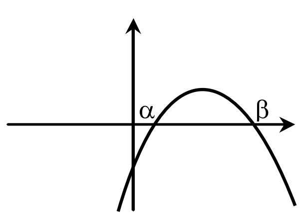
(ii) $a > 0$; $D = 0$



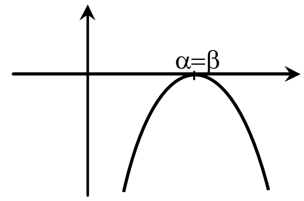
(iii) $a > 0$; $D < 0$



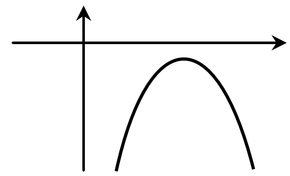
(iv) $a < 0$; $D > 0$



(v) $a < 0$; $D = 0$



(vi) $a < 0$; $D < 0$



(11) Plotting of Algebraic curve

From : $f(x) = (x - \alpha)^a (x - \beta)^b$ ...., $a, b \in N$.

Method :

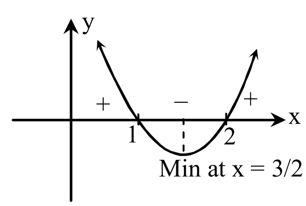
(i) Put $f(x) = 0$ to get change points.

(ii) Write these points on the number line.

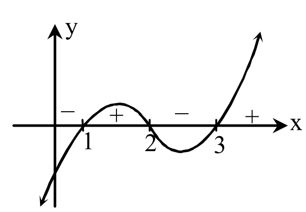
(iii) Write sign scheme of wavy curve method.

(iv) Wherever positive sign is occurring, the graph is above x-axis and wherever the sign is negative the graph is below x-axis.

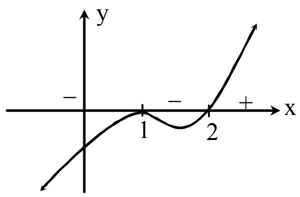
Ex. $y = (x - 1)(x - 2)$



Ex. $y = (x - 1)(x - 2)(x - 3)$



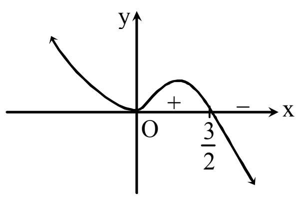
Ex. $y = (x-1)^2(x-2)$



Ex. $y = 3x^2 - 2x^3$

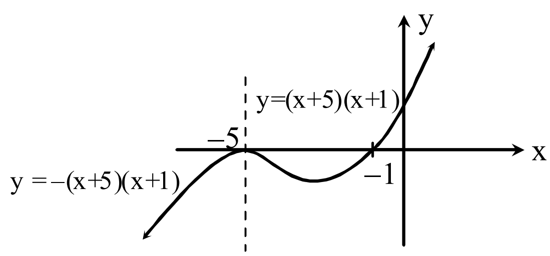
$\Rightarrow \quad y = x^2(3-2x)$

$\Rightarrow \quad y = -x^2(2x-3)$



Ex. $y = |x+5|\,(x+1)$

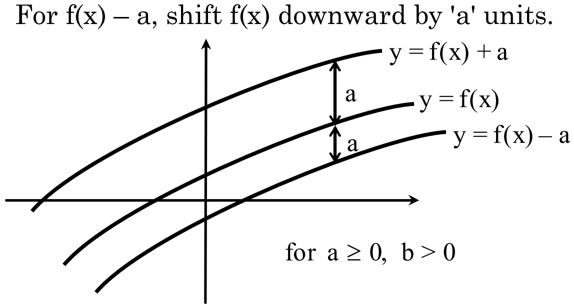
$$\Rightarrow \quad y = \begin{cases} (x+5)(x+1) & ; \quad x \geq -5 \\ -(x+5)(x+1) & ; \quad x < -5 \end{cases}$$



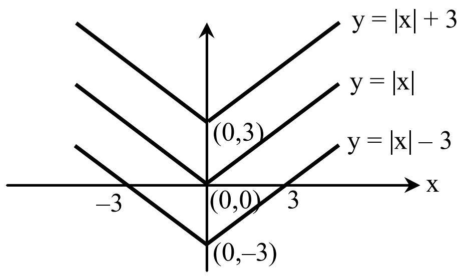
(12) To draw y = f(x) ± a from y = f(x)

For f(x) + a, shift f(x) upward by 'a' units.

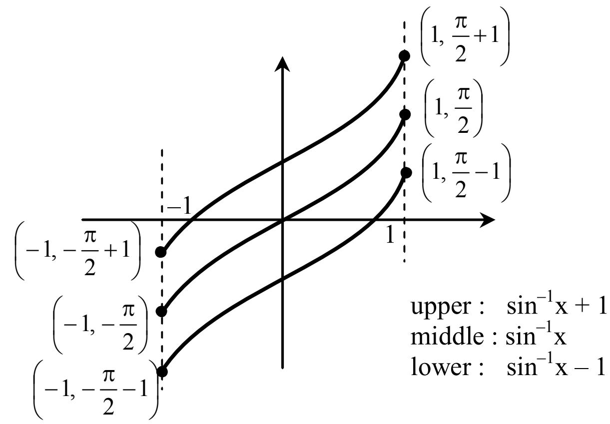
For f(x) – a, shift f(x) downward by 'a' units.



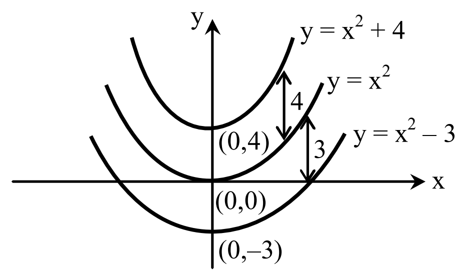
Ex. $y = |x| + 3$ and $y = |x| - 3$



Ex. $y = \sin^{-1}x + 1$ and $y = \sin^{-1}x - 1$



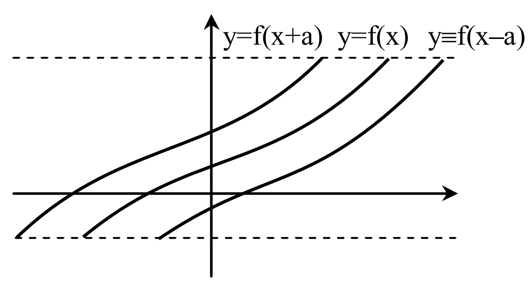
Ex. $y = x^2 + 4$ & $y = x^2 - 3$



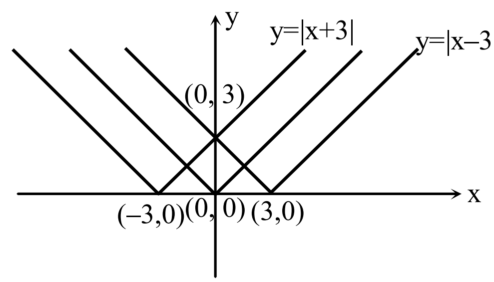
(13) To draw y = f(x ± a) from y = f(x)

For f(x + a) : shift y = f(x) left by 'a' units

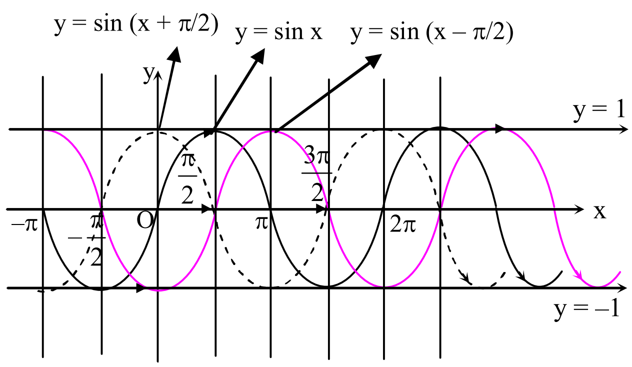
For f(x – a) : shift y = f(x) right by 'a' units



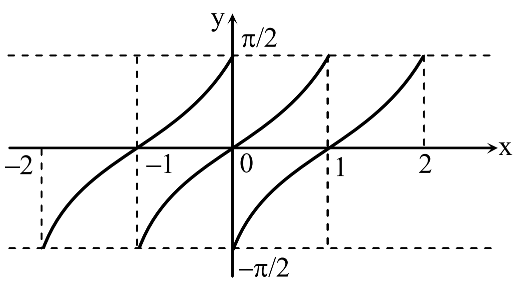
Ex. $y = |x-3|$ and $y = |x+3|$



Ex. $y = \sin\left(x - \frac{\pi}{2}\right)$ and $y = \sin\left(x + \frac{\pi}{2}\right)$



Ex. $y = \sin^{-1}(x - 1)$ and $y = \sin^{-1}(x + 1)$



Left : $\sin^{-1}(x + 1)$

Middle : $\sin^{-1}x$

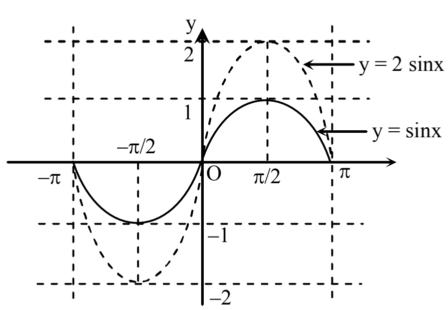
Right : $\sin^{-1}(x - 1)$

(14) To draw y = af(x) from y = f(x)

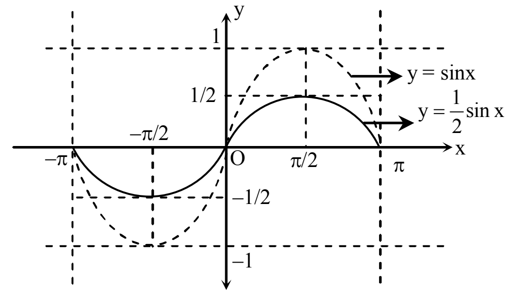
If $a > 1$, expand graph 'a' times along y-axis

If $a = \frac{1}{k}$, $k > 1$, contract graph 'k' times along y-axis

Ex. $y = \sin x$ and $y = 2\sin x$



Ex. $y = \sin x$ and $y = \frac{1}{2}\sin x$

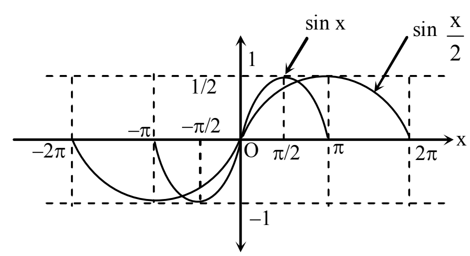


(15) To draw y = f(ax) from y = f(x)

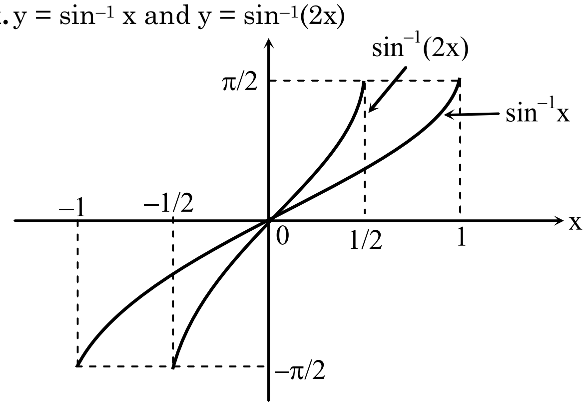
For $a > 1$, contract the graph of f(x) by 'a' times along x-axis.

For $a = \frac{1}{k}$, $k > 1$, expand the graph of f(x) by 'k' times along x-axis.

Ex. $y = \sin x$ and $y = \sin\frac{x}{2}$



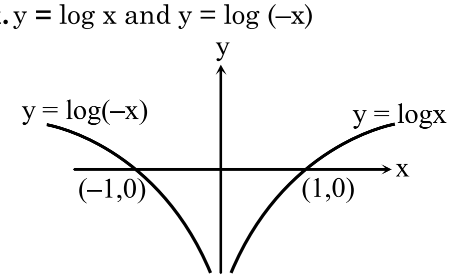
Ex. $y = \sin^{-1} x$ and $y = \sin^{-1}(2x)$



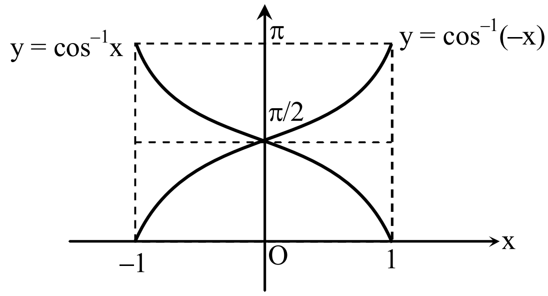
(16) To draw y = f(–x) from y = f(x)

Take mirror image of y = f(x) in y-axis

Ex. $y = \log x$ and $y = \log(-x)$



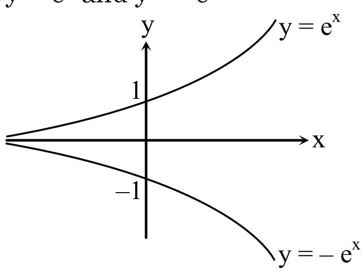
Ex. $y = \cos^{-1}x$ and $y = \cos^{-1}(-x)$



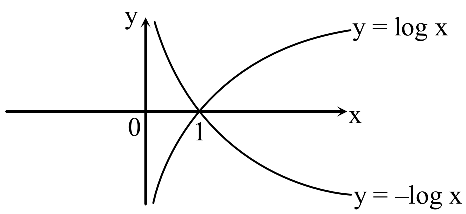
(17) To draw $y = -f(x)$ from $y = f(x)$

Take mirror image of $y = f(x)$ in the x-axis

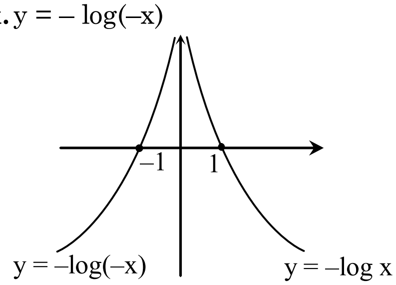
Ex. $y = e^x$ and $y = -e^x$



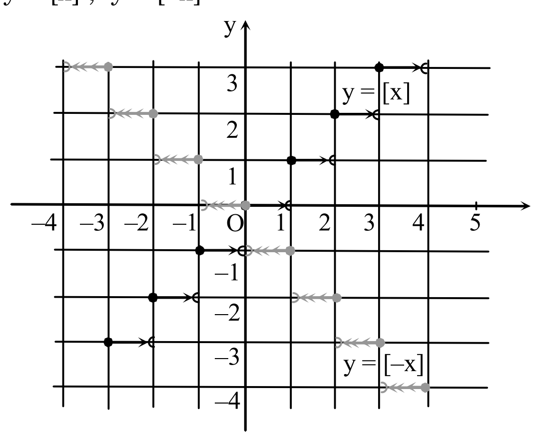
Ex. $y = \log x$ and $y = -\log x$



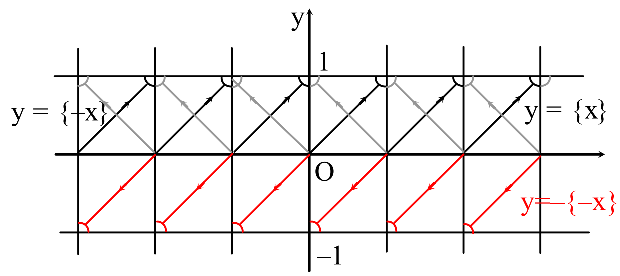
Ex. $y = -\log(-x)$



Ex. $y = [x]$, $y = [-x]$



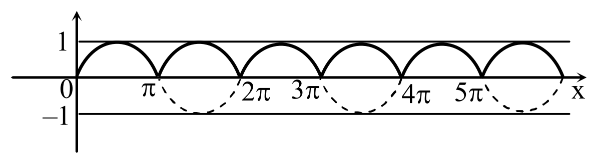
Ex. $y = -\{-x\}$, ({.} is F.P.F)



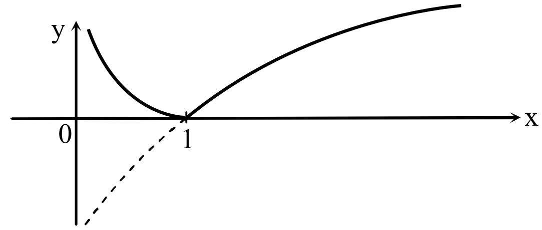
(18) To draw $y = |f(x)|$ from $y = f(x)$

1. Draw graph of f(x)
2. Take mirror image in x-axis for negative portion
3. Erase negative portion

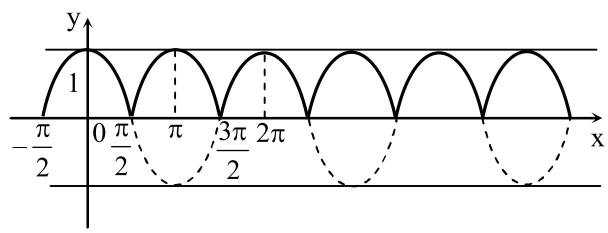
Ex. $y = |\sin x|$



Ex. $y = |\log x|$

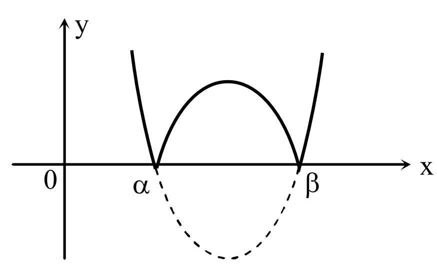


Ex. $y = |\cos x|$

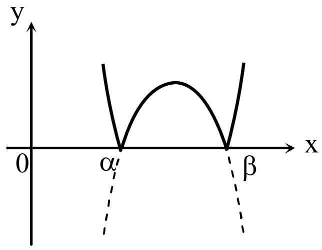


Ex. $y = |ax^2 + bx + c|$

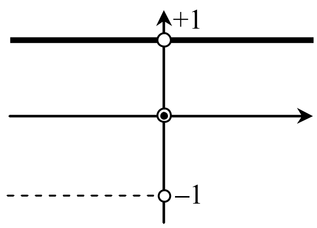
for $a > 0$; $D > 0$



for $a < 0; D > 0$



Ex. $y = |\operatorname{sgn}(x)|$

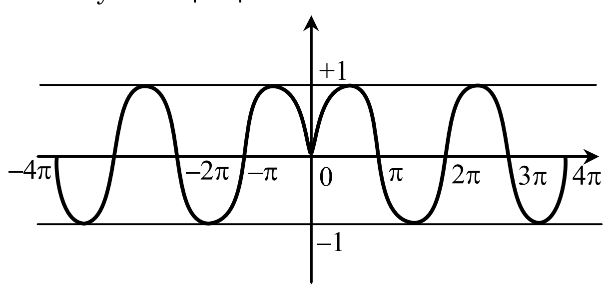


(19) To draw

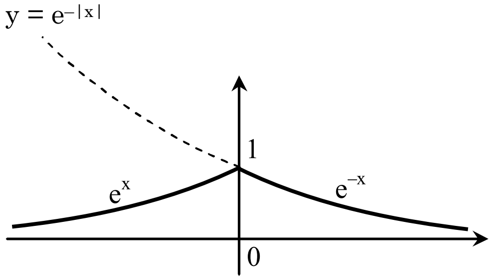
$y = f(|x|)$ from $y = f(x)$

1. Draw graph for positive values of x.
2. Take mirror image in y-axis.

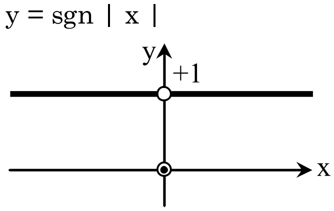
Ex. $y = \sin|x|$



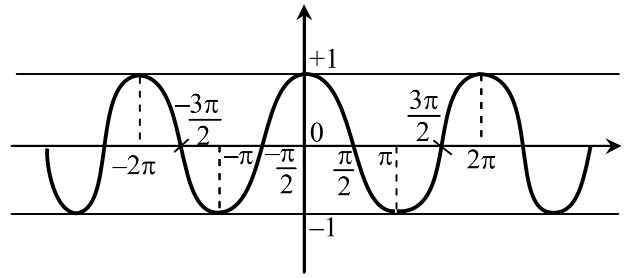
Ex. $y = e^{-|x|}$



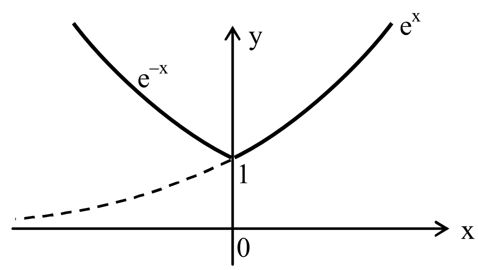
Ex. $y = \operatorname{sgn}|x|$



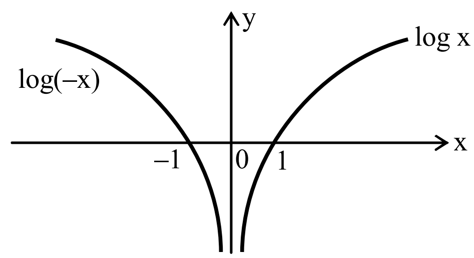
Ex. $y = \cos|x| = \cos x$



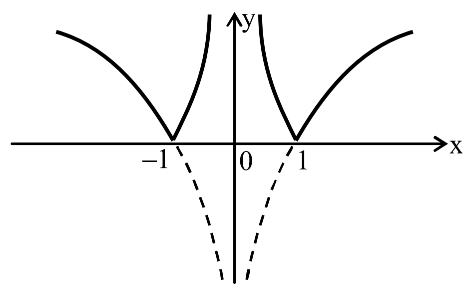
Ex. $y = e^{|x|}$



Ex. $y = \log|x|$



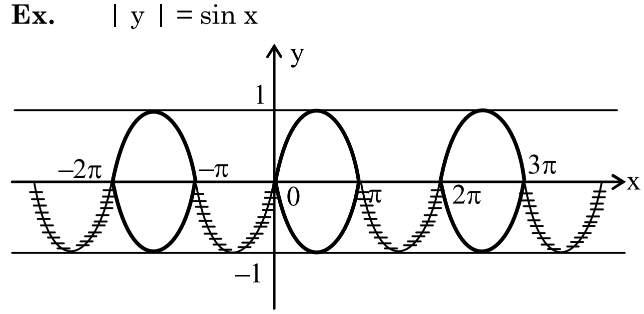
Ex. $y = |\log|x||$



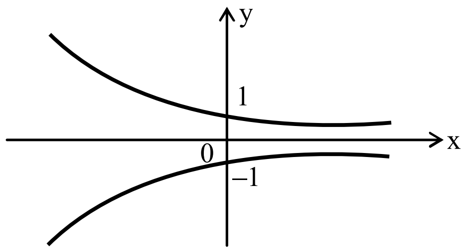
(20) To draw $|y| = f(x)$ from $y = f(x)$

1. Draw graph of f(x)
2. Eliminate negative portion
3. Take mirror image in x-axis for positive portion

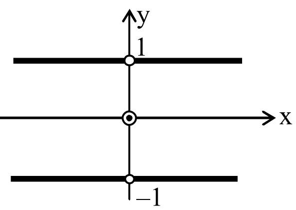
Ex. $|y| = \sin x$



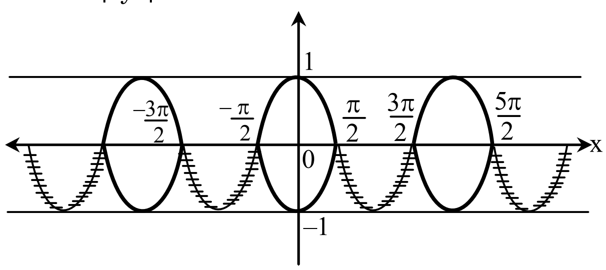
Ex. $|\,y\,| = e^{-x}$



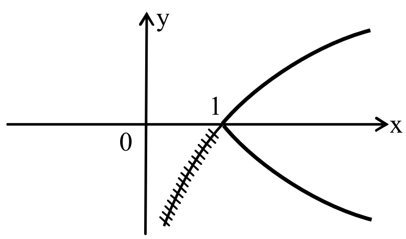
Ex. $|\,y\,| = |\,\text{sgn}\,|\,x\,|\,|$



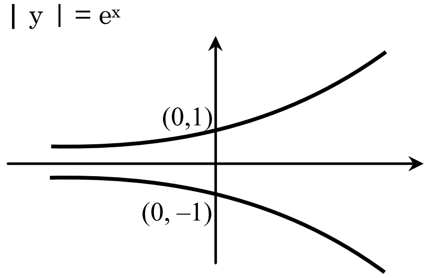
Ex. $|\,y\,| = \cos x$



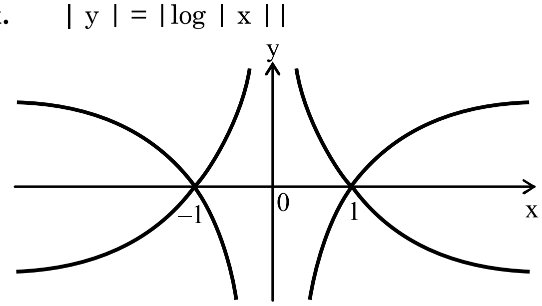
Ex. $|\,y\,| = \log\ x$



Ex. $|\,y\,| = e^{x}$



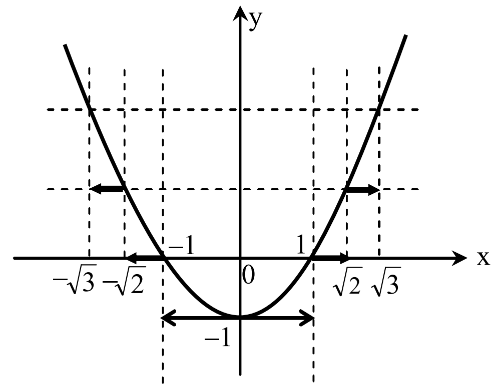
Ex. $|\,y\,| = |\log\,|\,x\,|\,|$



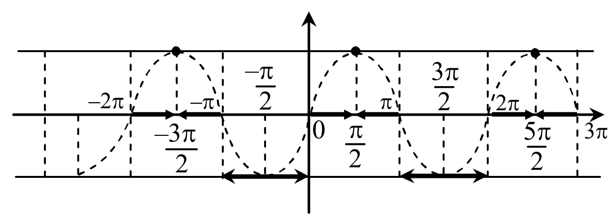
(21) To trace y = $[f(x)]$ from y = f(x) (where [.] is GIF)

1. Plot y = f(x)
2. Mark the interval of unit length with integers as end points on y-axis.
3. Mark the corresponding intervals on the x-axis.
4. Plot the value of [f(x)] for each intervals.

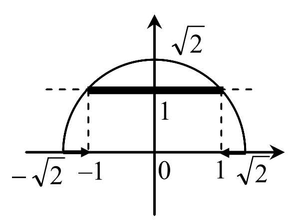
Ex. $y = [x^2 - 1]$ and $y = (x^2 - 1)$.



Ex. y = [sin x] and y = sin x.



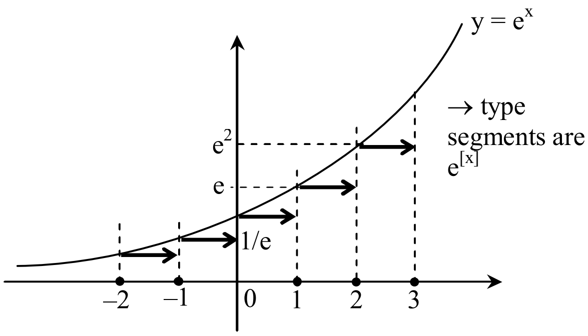
Ex. $y = [\sqrt{2 - x^2}\,]$



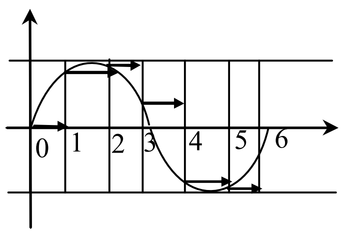
(22) To draw y = f [x] from y = f(x) (where [.] is GIF]

1. Plot lines parallel to y-axis for integral values of x.
2. Mark the points at integers on the curve
3. Take lower marked point of x say if $n < x < n + 1$, then take the point at $x = n$ and draw a horizontal line to nearest formed by $x = n + 1$

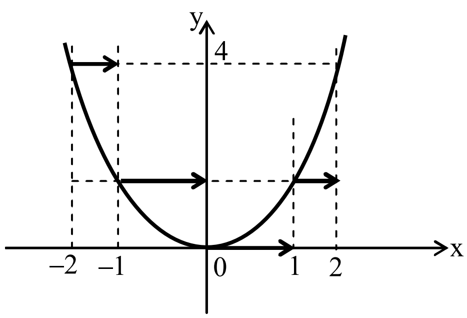
Ex. $y = e^x$ and $y = e^{[x]}$



Ex. $y = \sin x$ and $y = \sin [x]$



Ex. $y = x^2$ and $y = [x]^2$

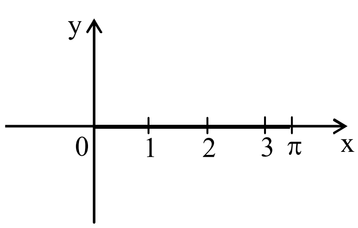


Ex. $y = [\sin[x]]$; $0 \le x \le \pi$

Look at the graph of $\sin[x]$, for $0 \le x \le \pi$.

All the segments are below 1 so $y = [\sin[x]] = 0$

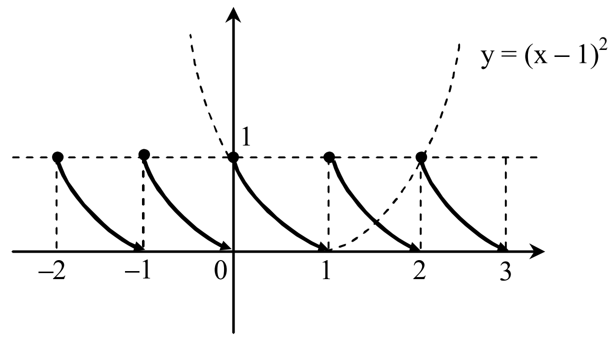
Thus



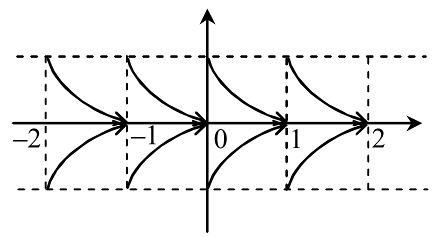
(23) To draw $y = f(\{x\})$ from $y = f(x)$

Retain the graph of $f(x)$ for values of $x \in [0, 1]$. It should be repeated from rest of unit intervals by looking periodicity 1.

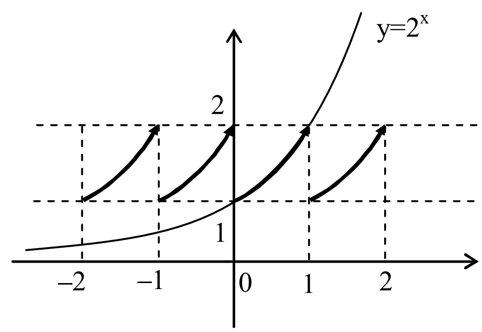
Ex. $y = (\{x\} - 1)^2$ and $y = (x - 1)^2$



Ex. $|\,y\,| = (\{x\} - 1)^2$



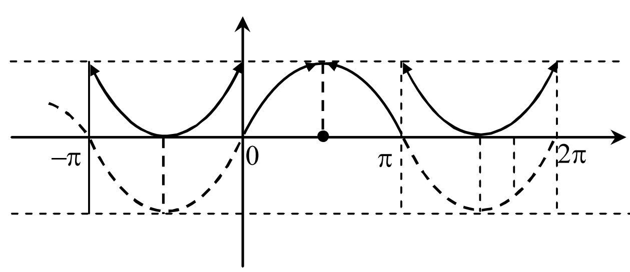
Ex. $y = 2^x/2^{[x]}$ i.e. $y = 2^{\{x\}}$



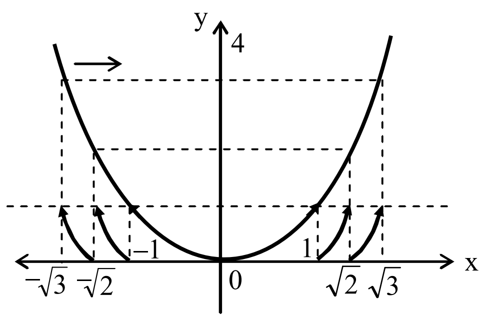
(24) To draw $y = \{f(x)\}$ from $y = f(x)$

Plot horizontal lines for all integral values of y and for the point of intersection on $y = f(x)$, plot vertical lines and translate the graph for $y = 0$ and $y = 1$

Ex. $y = \{\sin x\}$



Ex. $y = \{x^2\}$

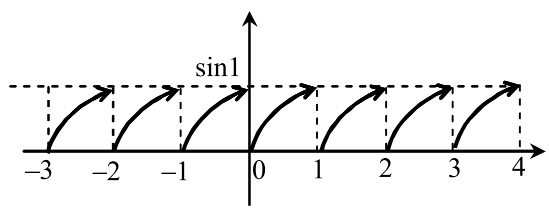


Ex. $y = \{\sin \{x\}\}$

As $\sin \{x\}$ is between $[0, \sin 1)$

$\because \sin 1 < 1$

So graph of $\sin \{x\}$ and $\{\sin \{x\}\}$ both are same.



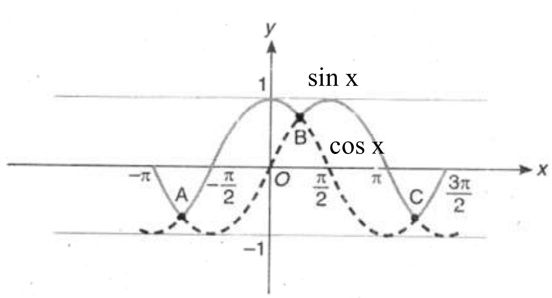
(25) To draw Max. or Min. {f(x).g(x)}

First draw both the curves f(x) and g(x). Observe their points of intersection. Among these points which one is above and which one is below.

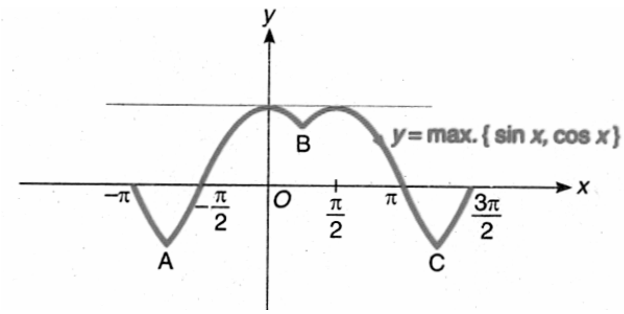
For max. take upper portions and for min take lower portions.

Ex. Sketch the graph of

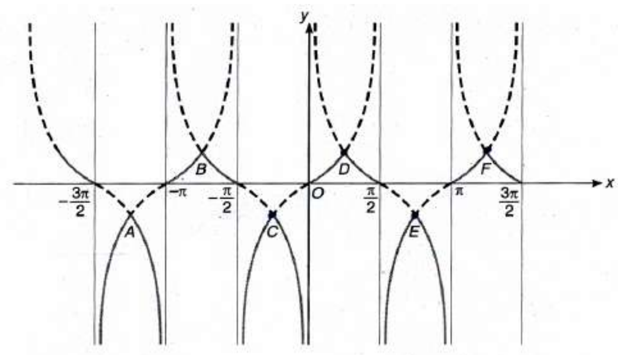
$y = \max. \{\sin x, \cos x\}, \forall\ x \in \left(-\pi, \dfrac{3\pi}{2}\right)$



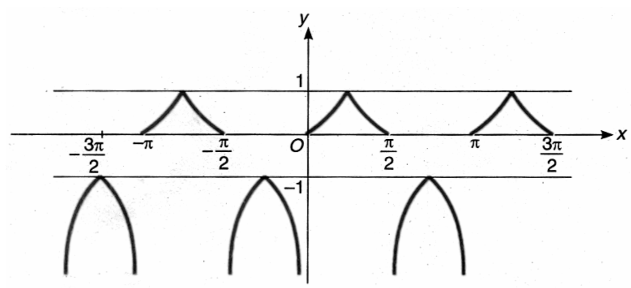
$\therefore$ **Graph of max {sin x, cos x}**



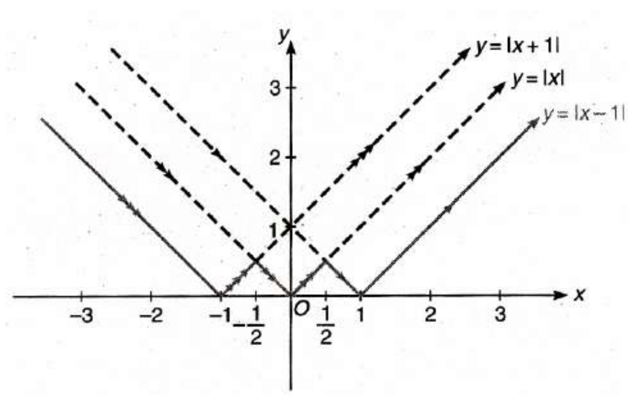
Ex. Sketch the graph for $y = \min\{\tan x, \cot x\}$



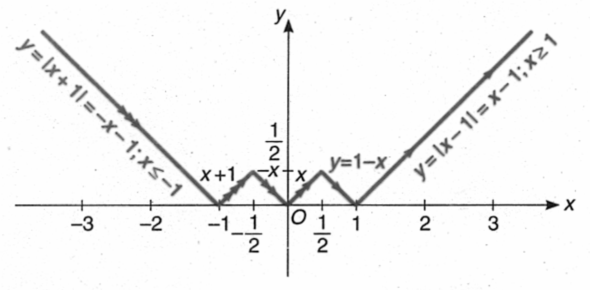
Graph of min {tan x, cot x}



Ex. Sketch the curve $y = \min\{|x|, |x - 1|, |x + 1|\}$

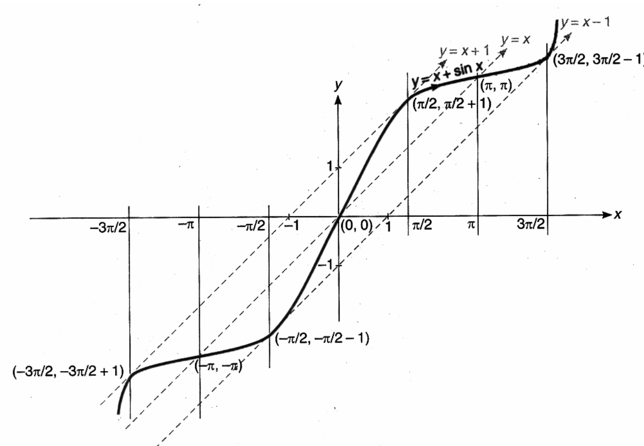


Graph for y = | x | , y = | x – 1|, y = |x + 1|

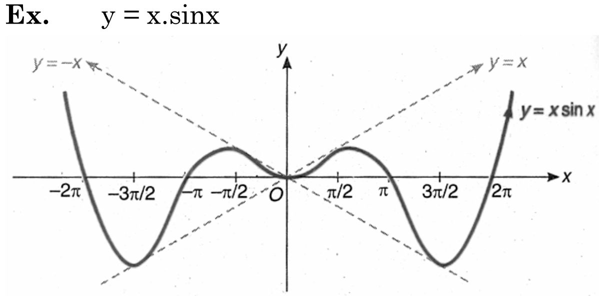


(26) To draw y = x + sin x and y = x.sinx

Ex. y = x + sin x



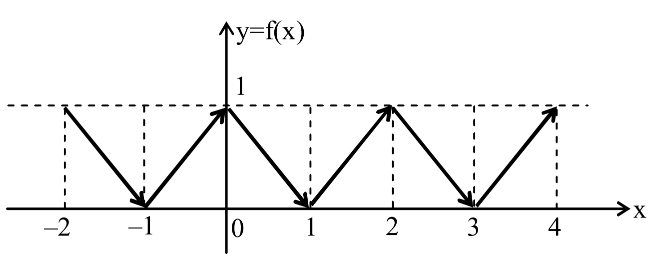
Ex. y = x.sinx



(27) To draw

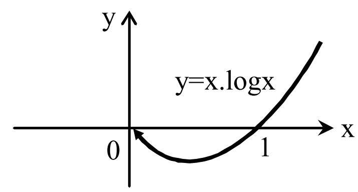
$$f(x) = \begin{cases} x-[x] & if\ \ [x]\ is\ odd \\ 1+[x]-x & if\ \ [x]\ is\ even \end{cases} \text{ i.e.}$$

$$f(x) = \begin{cases} \{x\} & if\ \ [x] = \pm 1, \pm 3.... \\ 1-\{x\} & if\ \ [x] = 0, \pm 2, \pm 4..... \end{cases}$$

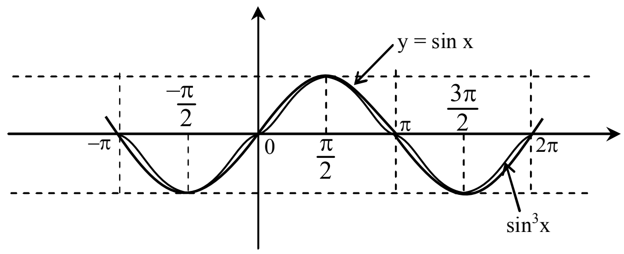


(28) To draw y = x log x

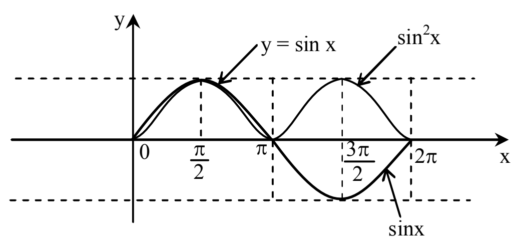
1. Take x > 0
2. $\lim_{x \to 0}$ x. log x → 0
3. x = 1, y = 0
4. y = x log x is continuous + differentiable for x > 0



(29) Graphs of $T^n(x)$, n ∈ N, like $\sin^2 x$, $\sin^3 x$ etc. $\sin^n x$. n ∈ N (odd natural)

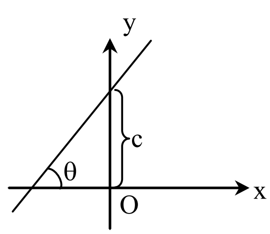


$\sin^n x$. n ∈ N (even natural)

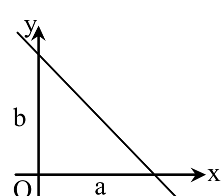


(30) Straight Line :

(i) **Slope and y-intercept form :**



(ii) **Double Intercept form :** $\frac{x}{a} + \frac{y}{b} = 1$



SOLVED EXAMPLES

Ex.1 Which of the following is a function?

(A) {(2, 1), (2, 2), (2, 3), (2, 4)}
(B) {(1, 4), (2, 5), (1, 6), (3, 9)}
(C) {(1, 2), (3, 3), (2, 3), (1, 4)}
(D) {(1, 2), (2, 2), (3, 2), (4, 2)}

Sol. We know that for a relation to be function every element of first set should be associated with one and only one element of second set but elements of first set can have same f-image in second set which is given in (D). **Ans.[D]**

Ex.2 If f (x) $= \frac{x}{x-1} = \frac{1}{y}$, then f (y) equals

(A) x (B) x – 1
(C) x + 1 (D) 1 – x

Sol. $f(y) = \frac{y}{y-1} = \frac{(x-1)/x}{\frac{x-1}{x}-1} = \frac{x-1}{x-1-x}$

$= 1 - x$ **Ans.[D]**

Ex.3 The domain of f(x) = $\frac{1}{x^3 - x}$ is -

(A) R – {–1,0,1} (B) R
(C) R – {0,1} (D) None of these

Sol. Domain = {x; x ∈R; $x^3 - x \neq 0$}
= R – {–1, 0, 1} **Ans.[A]**

Ex.4 The range of f (x) = cos $\frac{\pi[x]}{2}$ is -

(A) {0, 1} (B) {–1, 1}
(C) {–1, 0, 1} (D) [–1, 1]

Sol. [x] is an integer, cos (–x) = cos x and

$\cos\left(\frac{\pi}{2}\right) = 0$, $\cos 2\left(\frac{\pi}{2}\right) = -1$.

$\cos 0\left(\frac{\pi}{2}\right) = 1$, $\cos 3\left(\frac{\pi}{2}\right) = 0$,.....

Hence range = {–1, 0, 1) **Ans.[C]**

Ex.5 If f : $R^+ \to R^+$, f(x) = $x^2 + 2$ and
g : $R^+ \to R^+$, g(x) = $\sqrt{x+1}$
then (f + g) (x) equals -

(A) $\sqrt{x^2+3}$ (B) x + 3
(C) $\sqrt{x^2+2} + (x+1)$ (D) $x^2 + 2 + \sqrt{(x+1)}$

Sol. (f + g) (x) = f(x) + g(x)
$= x^2 + 2 + \sqrt{x+1}$ **Ans. [D]**

Ex.6 Function f(x) = $x^{-2} + x^{-3}$ is -

(A) a rational function
(B) an irrational function
(C) an inverse function
(D) None of these

Sol. f(x) = $\frac{1}{x^2} + \frac{1}{x^3} = \frac{x+1}{x^3}$

= ratio of two polynomials
∴ f(x) is a rational function. **Ans.[A]**

Ex.7 The period of | sin 2x | is-

(A) π/4 (B) π/2 (C) π (D) 2π

Sol. Here |sin 2x| = $\sqrt{\sin^2 2x}$

$= \sqrt{\frac{(1-\cos 4x)}{2}}$

Period of cos 4 x is π/2
Period of |sin 2x | will be π/2. **Ans.[B]**

Ex.8 If f(x) = $\frac{x-3}{x+1}$, then f [f {f (x)}] equals -

(A) x (B) 1/x (C) –x (D) –1/x

Sol. Here f {f(x)} = $f\left(\frac{x-3}{x+1}\right) = \frac{\left(\frac{x-3}{x+1}\right)-3}{\left(\frac{x-3}{x+1}\right)+1} = \frac{x+3}{1-x}$

∴ f [f {f (x)}] = $\frac{\frac{x+3}{1-x}-3}{\frac{x+3}{1-x}+1} = \frac{4x}{4} = x$ **Ans.[A]**

Ex.9 If f(x) = 2|x – 2| – 3|x – 3|, then the value of f(x) when 2 < x < 3 is -

(A) 5 – x (B) x – 5
(C) 5x – 13 (D) None of these

Sol. 2 < x < 3 ⇒ |x – 2| = x – 2
|x – 3| = 3 – x
f(x) = 2 (x – 2) – 3 (3 – x) = 5x – 13. **Ans.[C]**

Ex.10 Which of the following functions defined from R to R are one-one -

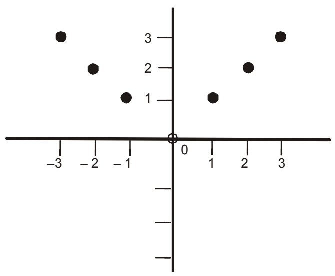
(A) f (x) = | x | (B) f (x) = cos x
(C) f (x) = e^x (D) f (x) = x^2

Sol. $x_1 \neq x_2 \Rightarrow e^{x_1} \neq e^{x_2}$
$\Rightarrow f(x_1) \neq f(x_2)$
$\therefore$ $f(x) = e^x$ is one-one. **Ans.[C]**

Ex.11 The function $f : R \to R$, $f(x) = x^2$ is -
(A) one-one but not onto
(B) onto but not one-one
(C) one-one onto
(D) None of these

Sol. $\because$ $4 \neq -4$, but $f(4) = f(-4) = 16$
$\therefore$ f is many one function.
Again $f(R) = R^+ \cup \{0\}$ R, therefore f is into.
Ans. [D]

Ex.12 If $f : I_0 \to N$, $f(x) = |x|$, then f is -



(A) one-one (B) onto
(C) one-one onto (D) none of these

Sol. Observing the graph of this function, we find that every line parallel to x-axis meets its graph at more than one point so it is not one-one. Now range of f = N = Co-domain, so it is onto. **Ans. [B]**

Ex.13 If $f : R - \{3\}$ $R - \{1\}$, $f(x) = \dfrac{x-2}{x-3}$ then function f(x) is -
(A) only one-one
(B) one-one into
(C) many one onto
(D) one-one onto

Sol. $\because$ $f(x) = \dfrac{x-2}{x-3}$

$\therefore$ $f'(x) = \dfrac{(x-3).1-(x-2).1}{(x-3)^2} = \dfrac{-1}{(x-3)^2}$

$\therefore$ $f'(x) < 0 \ \forall \ x \in R - \{3\}$
$\therefore$ f (x) is monotonically decreasing function
$\Rightarrow$ f is one-one function.
onto/ into : Let $y \in R - \{1\}$ (co-domain)
Then one element $x \in R - \{3\}$ is domain is such that

$f(x) = y \Rightarrow \dfrac{x-2}{x-3} = y$
$\Rightarrow x - 2 = xy - 3y$
$\Rightarrow x = \left(\dfrac{3y-2}{y-1}\right) = x \in R - \{3\}$
$\therefore$ the pre-image of each element of co-domain $R - \{1\}$ exists in domain $R - \{3\}$.
$\Rightarrow$ f is onto. **Ans. [D]**

Ex.14 Function $f : N \to N$, $f(x) = 2x + 3$ is -
(A) one-one onto
(B) one-one into
(C) many one onto
(D) many one into

Sol. f is one-one because for any $x_1, x_2 \in N$
$x_1 \neq x_2 \Rightarrow 2x_1 + 3 \neq 2x_2 + 3 \Rightarrow f(x_1) \neq f(x_2)$
Further $f^{-1}(x) = \dfrac{x-3}{2} \notin N$ (domain) when $x = 1, 2, 3$ etc.
$\therefore$ f is into which shows that f is one-one into.

__Alter__
$f(x) = 2x + 3$
$f'(x) = 2 > 0 \ \forall \ x \in N$
$\therefore$ f(x) is increasing function
$\therefore$ f(x) is one-one function
& $\because$ $x = 1, 2, 3, ...$
$\therefore$ min value of f(x) is $2.1 + 3 = 5$
$\therefore$ $f(x) \neq \{1, 2, 3, 4\}$
$\therefore$ Co Domain $\neq$ Range
$\therefore$ f(x) is into function. **Ans. [B]**

Ex.15 Function $f : R \to R$, $f(x) = x^3 - x$ is -
(A) one-one onto
(B) one-one into
(C) many-one onto
(D) many-one into

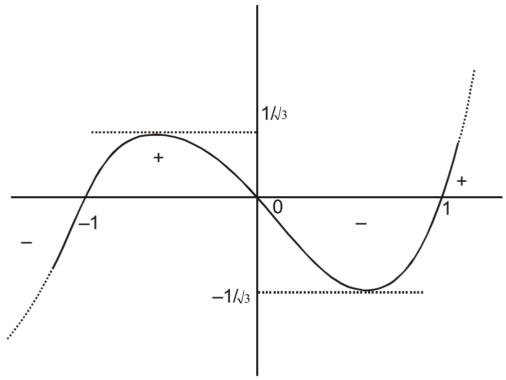
Sol. Since $-1 \neq 1$, but $f(-1) = f(1)$, therefore f is many-one.
Also let, $f(x) = x^3 - x = \alpha \Rightarrow x^3 - x - \alpha = 0$. This is a cubic equation in x which has at least one real root because complex roots always occur in pairs. Therefore each element of co-domain R has pre-image in R. Thus function f in onto .
$\therefore$ function f is many-one onto.

Alter

$f(x) = x^3 - x$

$= x(x-1)(x+1)$

graph of f(x) is



from graph function is many one- onto function **Ans. [C]**

Ex.16 If $f : R \to R$, $f(x) = 2x - 1$ and $g : R \to R$, $g(x) = x^2 + 2$, then (gof) (x) equals-

(A) $2x^2 - 1$ (B) $(2x-1)^2$

(C) $2x^2 + 3$ (D) $4x^2 - 4x + 3$

Sol. Here $(gof)(x) = g[f(x)] = g(2x-1)$

$= (2x-1)^2 + 2 = 4x^2 - 4x + 3$. **Ans. [D]**

Ex.17 If $f : R \to R$, $f(x) = 4x^3 + 3$, then $f^{-1}(x)$ equals-

(A) $\left(\frac{x-3}{4}\right)^{1/3}$ (B) $\left(\frac{x^{1/3}-3}{4}\right)$

(C) $\frac{1}{4}(x-3)^{1/3}$ (D) None of these

Sol. Since f is a bijection, therefore f^{-1} exists. Now if f-image of x is y, then $f^{-1}: R \to R$ defined as follows :

$f^{-1}(y) = x \Rightarrow f(x) = y$

But $f(x) = 4x^3 + 3 \Rightarrow y = 4x^3 + 3 \Rightarrow x = \left(\frac{y-3}{4}\right)^{1/3}$

Therefore $f^{-1}(y) = \left(\frac{y-3}{4}\right)^{1/3}$

$\Rightarrow f^{-1}(x) = \left(\frac{x-3}{4}\right)^{1/3}$ **Ans. [A]**

Ex.18 $f(x) = \sqrt{|x-1|}$ and $g(x) = \sin x$ then (fog) (x) equals -

(A) $\sin\left\{\sqrt{|x-1|}\right\}$ (B) $|\sin x/2 - \cos x/2|$

(C) $|\sin x - \cos x|$ (D) None of these

Sol. $(fog)(x) = f[g(x)] = f[\sin x]$

$= \sqrt{|\sin x - 1|}$

$= \sqrt{|1 - \sin x|}$

$= \sqrt{|\sin^2 x/2 + \cos^2 x/2 - 2\sin x/2 \cos x/2|}$

$= \sqrt{|(\sin x/2 - \cos x/2)^2|}$

$= |\sin x/2 - \cos x/2|$ **Ans.[B]**

Ex.19 If $f : R \to R$, $f(x) = 2x + 1$ and $g : R \to R$, $g(x) = x^3$, then $(gof)^{-1}(27)$ equals -

(A) –1 (B) 0

(C) 1 (D) 2

Sol. Here $f(x) = 2x + 1$ $f^{-1}(x) = \frac{x-1}{2}$

and $g(x) = x^3 \Rightarrow g^{-1}(x) = x^{1/3}$

$\because (gof)^{-1}(27) = (f^{-1}og^{-1})(27)$

$= f^{-1}[g^{-1}(27)] = f^{-1}[(27)^{1/3}]$

$= f^{-1}(3) = \frac{3-1}{2} = 1$ **Ans.[C]**

Ex.20 The domain of function $f(x) = \sqrt{2^x - 3^x}$ is -

(A) $(-\infty, 0]$ (B) R

(C) $[0, \infty)$ (D) No value of x

Sol. Domain $= \{x ; 2^x - 3^x \geq 0\} = \{x ; (2/3)^x \geq 1\}$

$= x \in (-\infty, 0]$ **Ans.[A]**

Ex.21 The domain of the function

$f(x) = \sin^{-1}\left(\log_2 \frac{x^2}{2}\right)$ is -

(A) $[-2, 2] - (-1, 1)$ (B) $[-1, 2] - \{0\}$

(C) $[1, 2]$ (D) $[-2, 2] - \{0\}$

Sol. We know that the domain of $\sin^{-1}x$ is $[-1,1]$. So for f(x) to be meaningful, we must have

$-1 \leq \log_2 \frac{x^2}{2} \leq 1$

$\Rightarrow 2^{-1} \leq x^2/2 \leq 2 \quad x \neq 0$

$\Rightarrow 1 \leq x^2 \leq 4, x \neq 0$

$\Rightarrow x \in [-2, -1] \cup [1, 2]$

$\Rightarrow x \in [-2, 2] - (-1, 1)$ **Ans.[A]**

Ex.22 The range of function $f(x) = \frac{x^2}{1+x^2}$ is -

(A) $R - \{1\}$ (B) $R^+ \cup \{0\}$

(C) $[0, 1]$ (D) None of these

Sol. Range is containing those real numbers y for which f(x) = y where x is real number.

Now f(x) = y = $\Rightarrow \dfrac{x^2}{1+x^2} = y$

$\Rightarrow$ x = $\sqrt{\dfrac{y}{(1-y)}}$ ....(1)

by (1) clearly y ≠ 1, and for x to be real

$\dfrac{y}{1-y} \geq 0 \Rightarrow y \geq 0$ and y < 1.

($\because$ If y = 2 then $\dfrac{y}{1-y} = \dfrac{2}{1-2} = (-2)$ and

$\sqrt{\dfrac{y}{(1-y)}} = \sqrt{-2} \notin R$)

$\therefore$ $0 \leq y < 1$

$\therefore$ Range of function = $(0 \leq y < 1) = [0, 1)$

Ans.[D]

Ex.23 If f(x) = cos (log x), then f(x) f(y) – 1/2 [f (x/y) + f(xy)] is equal to

(A) –1 (B) 1/2 (C) –2 (D) 0

Sol. cos (log x) cos (log y)

$-\dfrac{1}{2}$ [cos (log x/y) + cos (log xy)]

= $\dfrac{1}{2}$ [cos (log x + log y) + cos (log x – log y)]

– $\dfrac{1}{2}$ [cos (log x – log y) + cos (log x + log y)]

= 0 **Ans.[D]**

Ex.24 If f(x) = $\dfrac{2^x + 2^{-x}}{2}$, then f (x + y) . f (x – y) is equal to -

(A) $\dfrac{1}{2}$ [f (x+ y) + f(x – y)]

(B) $\dfrac{1}{2}$ [f (2x) + f (2y)]

(C) $\dfrac{1}{2}$ [f(x+ y). f(x – y)]

(D) None of these

Sol. f(x + y).f(x – y)

$= \dfrac{2^{x+y} + 2^{-x-y}}{2} \cdot \dfrac{2^{x+y} + 2^{-x-y}}{2}$

$= \dfrac{2^{2x} + 2^{2y} + 2^{-2x} + 2^{-2y}}{4}$

$= \dfrac{1}{2}\left[\dfrac{2^{2x} + 2^{-2x}}{2} \cdot \dfrac{2^{2y} + 2^{-2y}}{2}\right]$

= $\dfrac{1}{2}$ [f (2x) + f(2y)] **Ans.[B]**

Ex.25 If f : R → R. f (x) = 2x + |x| , then f (3x) – f (–x) – 4x equals -

(A) f(x) (B) – f(x)

(C) f (–x) (D) 2f(x)

Sol. f (3x) – f (–x) – 4x

= 6x + |3x| – {–2x + | –x|} – 4x

= 6x + 3 |x| + 2x – |x| – 4x

= 4x + 2 |x| = 2 f(x). **Ans.[D]**

Ex.26 If g(x) = x^2 + x – 2 & $\dfrac{1}{2}$ (gof) (x) = $2x^2$ – 5x + 2, then f(x) is equal to -

(A) 2x – 3

(B) 2x + 3

(C) $2x^2$ + 3x + 1

(D) $2x^2$ – 3x – 1

Sol. g(x) = x^2 + x – 2

$\Rightarrow$ (gof) (x) = g[f(x)] = $[f(x)]^2$ + f(x) – 2

Given, $\dfrac{1}{2}$ (gof) (x) = $2x^2$ – 5x + 2

$\therefore$ $\dfrac{1}{2}$ $[f(x)]^2$ + $\dfrac{1}{2}$ f(x) – 1 = $2x^2$ – 5x + 2

$\Rightarrow$ $[f(x)]^2$ + f(x) = 4 x^2 – 10x + 6

$\Rightarrow$ f(x) [f(x) + 1] = (2x – 3) [(2x – 3) + 1]

$\Rightarrow$ f(x) = 2x – 3 **Ans.[A]**

Ex.27 If f (x) = |x| and g(x) = [x], then value of fog $\left(-\dfrac{1}{4}\right)$ + gof $\left(-\dfrac{1}{4}\right)$ is -

(A) 0 (B) 1

(C) –1 (D) 1/4

Sol. fog = f $\left[g\left(-\dfrac{1}{4}\right)\right]$ f (–1) = 1

and gof $\left(-\dfrac{1}{4}\right)$ = g$\left[f\left(-\dfrac{1}{4}\right)\right]$ = g $\left(\dfrac{1}{4}\right)$ = [1/4] = 0

Required value = 1 + 0 = 1. **Ans.[B]**

EXERCISE - 1

Question based on Inequation

Q.1 The inequality $\frac{2}{x} < 3$ is true, when x belongs to-

(A) $\left[\frac{2}{3},\infty\right)$ (B) $\left(-\infty,\frac{2}{3}\right]$

(C) $\left(\frac{2}{3},\infty\right) \cup (-\infty, 0)$ (D) none of these

Q.2 $\frac{x+4}{x-3} < 2$ is satisfied when x satisfies-

(A) $(-\infty, 3) \cup (10, \infty)$

(B) $(3, 10)$

(C) $(-\infty, 3) \cup [10, \infty)$

(D) none of these

Q.3 Solution of $\frac{2x-3}{3x-5} \ge 3$ is -

(A) $\left[1,\frac{12}{7}\right)$ (B) $\left(\frac{5}{3},\frac{12}{7}\right]$

(C) $\left(-\infty,\frac{5}{3}\right)$ (D) $\left[\frac{12}{7},\infty\right)$

Q.4 Solution of $(x-1)^2 (x+4) < 0$ is-

(A) $(-\infty, 1)$ (B) $(-\infty, -4)$

(C) $(-1, 4)$ (D) $(1, 4)$

Q.5 Solution of $(2x + 1)(x - 3)(x + 7) < 0$ is-

(A) $(-\infty, -7) \cup \left(-\frac{1}{2},3\right)$

(B) $(-\infty, -7) \cup \left(\frac{1}{2},3\right)$

(C) $(-\infty, 7) \cup \left(-\frac{1}{2},3\right)$

(D) $(-\infty, -7) \cup (3, \infty)$

Q.6 If $x^2 + 6x - 27 > 0$ and $x^2 - 3x - 4 < 0$, then-

(A) $x > 3$ (B) $x < 4$

(C) $3 < x < 4$ (D) $x = \frac{7}{2}$

Q.7 If $x^2 - 1 \le 0$ and $x^2 - x - 2 \ge 0$, then x line in the interval/set

(A) $(-1, 2)$ (B) $(-1, 1)$

(C) $(1, 2)$ (D) $\{-1\}$

Question based on Definition of function

Q.8 Which of the following relation is a function ?

(A) $\{(1, 4), (2, 6), (1, 5), (3, 9)\}$

(B) $\{(3, 3), (2, 1), (1, 2), (2, 3)\}$

(C) $\{(1, 2), (2, 2,), (3, 2), (4, 2)\}$

(D) $\{(3, 1), (3, 2), (3, 3), (3, 4)\}$

Q.9 If $x, y \in R$, then which of the following rules is not a function-

(A) $y = 9 - x^2$ (B) $y = 2x^2$

(C) $x^2 + y^2 = 1$ (D) $y = x^2 + 1$

Question based on Even and Odd function

Q.10 Which one of the following is not an odd function-

(A) sin x (B) tan x

(C) $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ (D) None of these

Q.11 The function $f(x) = \frac{\sin^4 x + \cos^4 x}{x + \tan x}$ is -

(A) odd

(B) Even

(C) neither even nor odd

(D) odd and periodic

Q.12 $f(x) = \cos \log (x + \sqrt{1+x^2})$ is

(A) even function

(B) odd function

(C) neither even nor odd

(D) constant

Q.13 A function whose graph is symmetrical about the y-axis is given by-

(A) $f(x) = \log_e (x + \sqrt{x^2+1})$

(B) $f(x) = x + x^3$

(C) $f(x) = \cos x + \sin x$

(D) None of these

Q.14 Which of the following is an even function?

(A) $x\frac{a^x - 1}{a^x + 1}$ (B) tan x

(C) $\frac{a^x - a^{-x}}{2}$ (D) $\frac{a^x + 1}{a^x - 1}$

Q.15 In the following, odd function is -
(A) $\cos x^2$ (B) $(e^x + 1)/(e^x - 1)$
(C) $x^2 - |x|$ (D) None of these

Q.16 The function $f(x) = x^2 - |x|$ is-
(A) an odd function
(B) a rational function
(C) an even function
(D) None of these

Question based on Periodic function

Q.17 The period of $\sin^4 x + \cos^4 x$ is -
(A) π (B) $\pi/2$
(C) 2π (D) None of these

Q.18 The period of function $|\cos 2x|$ is -
(A) π (B) $\pi/2$ (C) 4π (D) 2π

Q.19 The period of function $\sin\left(\frac{\pi x}{2}\right) + \cos\left(\frac{\pi x}{2}\right)$ is-
(A) 4 (B) 6 (C) 12 (D) 24

Q.20 The period of the function $f(x) = \log \cos 2x + \tan 4x$ is-
(A) $\pi/2$ (B) π (C) 2π (D) $2\pi/5$

Q.21 The period of the function $f(x) = 2\cos\frac{1}{3}(x-\pi)$ is -
(A) 6π (B) 4π (C) 2π (D) π

Q.22 In the following which function is not periodic-
(A) $\tan 4x$ (B) $\cos 2\pi x$
(C) $\cos x^2$ (D) $\cos^2 x$

Question based on Domain, Co-domain and Range of function

Q.23 Domain of the function $f(x) = \frac{1}{\sqrt{x+2}}$ is-
(A) R (B) $(-2, \infty)$
(C) $[2, \infty]$ (D) $[0, \infty]$

Q.24 $f(x) = \sqrt{\sin x - \cos x}$
(A) $\left[2n\pi + \frac{\pi}{4}, 2n\pi + \frac{5\pi}{4}\right]$
(B) $\left[2n\pi - \frac{\pi}{4}, 2n\pi + \frac{5\pi}{4}\right]$
(C) $\left[2n\pi + \frac{\pi}{4}, 2n\pi - \frac{5\pi}{4}\right]$
(D) none

Q.25 The domain of the function $\log\sqrt{\frac{3-x}{2}}$ is-
(A) $(3, \infty)$ (B) $(-\infty, 3)$
(C) $(0, 3)$ (D) $(-3, 3)$

Q.26 Domain of the function $\cos^{-1}(4x - 1)$ is-
(A) $(0, 1/2)$ (B) $[0, 1/2]$
(C) $[1/2, 2]$ (D) None of these

Q.27 Domain of the function $\log|x^2 - 9|$ is-
(A) R (B) $R - [-3, 3]$
(C) $R - \{-3, 3\}$ (D) None of these

Q.28 The domain of the function- $f(x) = \sqrt{x-1} + \sqrt{6-x}$ is-
(A) $(1, 6)$ (B) $[1, 6]$
(C) $[1, \infty)$ (D) $(-\infty, 6]$

Q.29 The domain of the function $f(x) = \sqrt{(2-2x-x^2)}$ is -
(A) $-\sqrt{3} \le x \le \sqrt{3}$
(B) $-1-\sqrt{3} \le x \le -1+\sqrt{3}$
(C) $-2 \le x \le 2$
(D) $-2+\sqrt{3} \le x \le -2-\sqrt{3}$

Q.30 Domain of a function $f(x) = \sin^{-1} 5x$ is-
(A) $\left(-\frac{1}{5}, \frac{1}{5}\right)$ (B) $\left[-\frac{1}{5}, \frac{1}{5}\right]$
(C) R (D) $\left(0, \frac{1}{5}\right)$

Q.31 The range of the function $f : R \to R$, $f(x) = \tan^{-1} x$ is-
(A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (B) $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$
(C) R (D) None of these

Q.32 The range of $f(x) = \sin\frac{\pi}{2}[x]$ is, where [.] represents G.I.F. -
(A) $\{-1, 1\}$ (B) $\{-1, 0, 1\}$
(C) $\{0, 1\}$ (D) $[-1, 1]$

Q.33 Domain and range of $f(x) = \frac{|x-3|}{x-3}$ are respectively-
(A) R, [–1, 1] (B) R – {3}, {1, –1}
(C) R^+, R (D) None of these

Q.34 The domain of the function f(x) = sin 1/x is -
(A) R (B) R^+ (C) R_0 (D) R^-

Q.35 Range of the function f(x) = 9 – 7 sin x is-
(A) (2, 16) (B) [2, 16]
(C) [–1, 1] (D) (2, 16]

Q.36 For real values of x, range of function
$y = \frac{1}{2-\sin 3x}$ is -
(A) $\frac{1}{3} \le y \le 1$ (B) $-\frac{1}{3} \le y \le 1$
(C) $-\frac{1}{3} > y > -1$ (D) $\frac{1}{3} > y > 1$

Q.37 If f : R → R, $f(x) = \begin{cases} 1, & \text{when } x \in Q \\ -1, & \text{when } x \notin Q \end{cases}$, then image set of R under f is -
(A) {1, 1} (B) (–1, –1)
(C) {1, –1} (D) None of these

Q.38 If f : R→R, f(x) = x^2, then {x | f(x) = –1} equals-
(A) {–1, 1} (B) {1}
(C) ϕ (D) None of these

Q.39 The range of f(x) = cos 2x – sin 2x contains the set -
(A) [2, 4] (B) [–1, 1]
(C) [–2, 2] (D) [–4, 4]

Q.40 If the domain of the function $f(x) = \frac{|x|}{x}$ be [3, 7] then its range is-
(A) [–1, 1] (B) {–1, 1}
(C) {1} (D) {–1}

Q.41 The domain of the function
$f(x) = \frac{1}{\sqrt{x-[x]}}$ is, where [.] represents G.I.F. -
(A) R (B) R–Z
(C) Z (D) None of these

Q.42 The range of the function f(x) = 2 + x – [x–3] is, where [.] represents G.I.F. -
(A) [5, 6] (B) [5, 6)
(C) R (D) None of these

Question based on

Value of function

Q.43 Let $f\left(x+\frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ $(x \ne 0)$, then f(x) equals -
(A) $x^2 - 2$ (B) $x^2 - 1$
(C) x^2 (D) None of these

Q.44 If f : R → R, $f(x) = \begin{cases} 1, & \text{when } x \in Q \\ -1, & \text{when } x \notin Q \end{cases}$, then which of the following statement is wrong ?
(A) $f(\sqrt{2}) = -1$ (B) $f(\pi) = -1$
(C) f(e) = 1 (D) $f(\sqrt{4}) = 1$

Q.45 If $f(x) = \frac{x(x-1)}{2}$, then the value of f (x + 2) is-
(A) f(x) + f(x + 1) (B) $\frac{x+2}{x}f(x+1)$
(C) $\frac{(x+1)}{2}f(x+1)$ (D) $\frac{(x+2)}{2}f(x+1)$

Q.46 If f(x + ay, x – ay) = axy, then f(x, y) equals-
(A) $\frac{x^2+y^2}{4}$ (B) $\frac{x^2-y^2}{4}$
(C) x^2 (D) y^2

Q.47 If f(x) = cos (log x), then $\frac{f(xy)+f(x/y)}{f(x)f(y)}$ equals-
(A) 1 (B) –1 (C) 0 (D) 2

Q.48 If f(x) = |x| + |x – 1|, then for 0 < x < 1, f(x) equals-
(A) 1 (B) –1
(C) 2x + 1 (D) 2x – 1

Q.49 f(2x + 3y, 2x – 7y) = 20 x then f(x , y) equals to –
(A) 7x – 3y (B) 7x + 3y
(C) 3x – 7y (D) x – 10y

Q.50 If $f(x) = \log_a x$, then f(ax) equals-
(A) f(a) f(x) (B) 1+ f(x)
(C) f(x) (D) a f(x)

Q.51 If f(x) = (ax – c)/(cx – a) = y, then f(y) equals-
(A) x (B) 1/x (C) 1 (D) 0

Question based on Mapping

Q.52 If $f : I \to I, f(x) = x^3 + 1$, then *f* is -
(A) one-one but not onto
(B) onto but not one-one
(C) One-one onto
(D) None of these

Q.53 Function $f : R \to R$, $f(x) = x\,|x|$ is -
(A) one-one but not onto
(B) onto but not one-one
(C) one-one onto
(D) neither one-one nor onto

Q.54 $f : R \to R$, $f(x) = \frac{x^2}{1+x^2}$, is -
(A) many-one function
(B) odd function
(C) one-one function
(D) None of these

Q.55 If $f : R_0 \to R_0$, $f(x) = \frac{1}{x}$, then f is -
(A) one-one but not onto
(B) onto but not one-one
(C) neither one-one nor onto
(D) both one-one and onto

Q.56 Function $f : R \to R$, $f(x) = x + |x|$ is-
(A) one-one (B) onto
(C) one-one onto (D) None of these

Q.57 Function $f : \left]\frac{\pi}{2}, \frac{3\pi}{2}\right[\to R$, f(x) = tan x is-
(A) one-one (B) onto
(C) one-one onto (D) None of these

Q.58 Function $f : \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \to [-1,1]$, f(x) = sin x is -
(A) one-one (B) onto
(C) one-one onto (D) None of these

Q.59 $f : N \to N$ where $f(x) = x - (-1)^x$ then f is -
(A) one-one and into
(B) many-one and into
(C) one-one and onto
(D) many-one and onto

Q.60 If $f : R \to R$, $f(x) = e^x + e^{-x}$, then f is -
(A) one-one but not onto
(B) onto but not one-one
(C) neither one-one nor onto
(D) both one-one and onto

Q.61 If $f : R \to [-1, 1]$, f(x) = sin x, then f is-
(A) one-one onto (B) one-one into
(C) many-one onto (D) many-one into

Q.62 If $f : R \to R$, $f(x) = \sin^2 x + \cos^2 x$, then f is-
(A) one-one but not onto
(B) onto but not one-one
(C) neither one-one nor onto
(D) both one-one onto

Q.63 Which of the following functions from Z to itself are bijections ?
(A) $f(x) = x^3$ (B) f(x) = x + 2
(C) f(x) = 2x + 1 (D) $f(x) = x^2 + x$

Q.64 Which of the following functions from $A = \{x: -1 \le x \le 1\}$ to itself are bijections ?
(A) $f(x) = \frac{x}{2}$ (B) $g(x) = \sin\left(\frac{\pi x}{2}\right)$
(C) $h(x) = |x|$ (D) $k(x) = x^2$

Q.65 Which of the following function is onto ?
(A) $f : R \to R$; $f(x) = 3^x$
(B) $f : R \to R^+$; $f(x) = e^{-x}$
(C) f: $[0, \pi/2] \to [-1, 1]$; f(x) = sin x
(D) None of these

Q.66 Which of the following function defined from R to R is onto ?
(A) $f(x) = |x|$ (B) $f(x) = e^{-x}$
(C) $f(x) = x^3$ (D) f(x) = sin x.

Q.67 If $f : I \to I$, $f(x) = x^2 - x$, then f is -
(A) one-one onto
(B) one-one into
(C) many-one onto
(D) many-one into

Question based on Composite function

Q.68 If f(x) = 2x and g is identity function, then-
(A) (fog) (x) = g(x)
(B) (g + g) (x) = g(x)
(C) (fog) (x) = (g + g) (x)
(D) None of these

Q.69 gof exists, when-
(A) domain of f = domain of g
(B) co-domain of f = domain of g
(C) co-domain of g = domain of g
(D) co-domain of g = co-domain of f

Q.70 If $f : R \to R$, $f(x) = x^2 + 2x - 3$ and $g : R \to R$, $g(x) = 3x - 4$, then the value of fog (x) is-
(A) $3x^2 + 6x - 13$
(B) $9x^2 - 18x + 5$
(C) $(3x - 4)^2 + 2x - 3$
(D) None of these

Q.71 If $f : R \to R$, $f(x) = x^2 - 5x + 4$ and $g : R \to R$, $g(x) = \log x$, then the value of (gof) (2) is -
(A) 0 (B) ∞
(C) $-\infty$ (D) Undefined

Q.72 If $f : R^+ \to R^+$, $f(x) = x^2 + 1/x^2$ and $g : R^+ \to R^+$, $g(x) = e^x$ then (gof) (x) equals-
(A) $e^{x^2} + e^{x^{-2}}$ (B) $e^{x^2} + \frac{1}{e^{x^{-2}}}$
(C) $e^{2x} + e^{-2x}$ (D) $e^{x^2} . e^{x^{-2}}$

Q.73 If $f : R \to R$, $g : R \to R$ and $f(x) = 3x + 4$ and $(gof)(x) = 2x - 1$, then the value of g(x) is-
(A) $2x - 1$ (B) $2x - 11$
(C) $\frac{1}{3}(2x - 11)$ (D) None of these

Q.74 If $f : R \to R$, $g : R \to R$ and $g(x) = x + 3$ and $(fog)(x) = (x + 3)^2$, then the value of f(–3) is-
(A) –9 (B) 0
(C) 9 (D) None of these

Q.75 If f(x) = ax + b and g(x) = cx + d, then f(g(x)) = g(f(x)) is equivalent to-
(A) f(a) = g(c) (B) f(b) = g(b)
(C) f(d) = g(b) (D) f(c) = g(a)

Q.76 If $f:[0,1] \to [0,1]$, $f(x) = \frac{1-x}{1+x}$. $g : [0,1] \to [0,1]$, $g(x) = 4x(1 - x)$, then (fog) (x) equals-
(A) $\frac{1-4x+4x^2}{1+4x-4x^2}$ (B) $\frac{8x(1-x)}{(1+x)^2}$
(C) $\frac{1-4x-4x^2}{1+4x-4x^2}$ (D) None of these

Q.77 If f, g, h are three functions in any set, then wrong statement is -
(A) $(fog)^{-1} = g^{-1}of^{-1}$
(B) gof $\neq$ fog
(C) (fog)oh = fo(goh)
(D) $(gof)^{-1} = g^{-1}of^{-1}$

Q.78 If $f(x) = \frac{1-x}{1+x}$, then f [f (sinθ)] equals -
(A) sin θ (B) tan (θ/2)
(C) cot (θ/2) (D) cosec θ

Q.79 If $f(x) = (a - x^n)^{1/n}$, $n \in N$, then f [f(x)] =
(A) 0 (B) x
(C) x^n (D) $(a^n - x)^n$

Q.80 If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \left(\frac{3x+x^3}{1+3x^2}\right)$, then f[g(x)] is equal to-
(A) –f(x) (B) 3f(x)
(C) $[f(x)]^3$ (D) None of these

Q.81 If function $f(x) = \begin{cases} 1, & \text{when } x \in Q \\ 0, & \text{when } x \notin Q \end{cases}$, (fof) ($\sqrt{4}$) the value will be-
(A) 0 (B) 2
(C) 1 (D) None of these

Q.82 If $f(y) = \frac{y}{\sqrt{1-y^2}}$, $g(y) = \frac{y}{\sqrt{1+y^2}}$, then (fog)(y) equals -
(A) $\frac{y}{\sqrt{1-y^2}}$ (B) $\frac{y}{\sqrt{1+y^2}}$
(C) y (D) $\frac{1-y^2}{1+y^2}$

Q.83 If f(x) = [x] and g(x) = cos (πx), then the range of gof is -
(A) {0} (B) {–1, 1}
(C) {–1, 0, 1} (D) [–1, 1]

Question based on Inverse function

Q.84 If f : R→ R, f(x) = $x^2 + 3$, then pre-image of 2 under f is -
(A) {1,–1} (B) {1} (C) {–1} (D) ϕ

Q.85 Which of the following functions has its inverse-
(A) f : R →R, f(x) = a^x
(B) f : R→R, f(x) = |x| + |x – 1|
(C) f : R_0 →R^+, f(x) = |x|
(D) f : [π, 2π]→ [–1,1], f(x) = cos x

Q.86 If function f : R→R^+, f(x) = 2^x, then f^{-1} (x) will be equal to-
(A) $\log_x 2$ (B) $\log_2 (1/x)$
(C) $\log_2 x$ (D) None of these

Q.87 The inverse of the function
$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ + 2 is given by -
(A) $\log\left(\frac{x-2}{x-1}\right)^{1/2}$ (B) $\log\left(\frac{x-1}{x+1}\right)^{1/2}$
(C) $\log\left(\frac{x}{2-x}\right)^{1/2}$ (D) $\log\left(\frac{x-1}{3-x}\right)^{1/2}$

Q.88 If f : [1, ∞) → [2, ∞) is given by $f(x) = x + \frac{1}{x}$ then $f^{-1}(x)$ equals -
(A) $\frac{x+\sqrt{x^2-4}}{2}$ (B) $\frac{x}{1+x^2}$
(C) $\frac{x-\sqrt{x^2-4}}{2}$ (D) $1 + \sqrt{x^2-4}$

Q.89 If f(x) = $\log_e(x + \sqrt{1+x^2})$, then $f^{-1}(x)$ equals-
(A) $\log (x - \sqrt{1+x^2})$ (B) $\frac{e^x + e^{-x}}{2}$
(C) $\frac{e^x - e^{-x}}{2}$ (D) $\frac{e^x - e^{-x}}{e^x + e^{-x}}$

Q.90 If f(x) = $x^3 - 1$ and domain of f = {0, 1, 2, 3}, then domain of f^{-1} is -
(A) {0, 1, 2, 3} (B) {1, 0, –7, –26}
(C) {–1, 0, 7, 26} (D) {0, –1, –2, –3}

Q.91 If f(x) = $\{4 - (x - 7)^3\}^{1/5}$, then its inverse is-
(A) $7 - (4 - x^5)^{1/3}$ (B) $7 - (4 + x^5)^{1/3}$
(C) $7 + (4 - x^5)^{1/3}$ (D) None of these

EXERCISE -2

Q.1 The range of f (x) = $\sin^{-1}\sqrt{x^2+x+1}$ is -
(A) (0, π/2] (B) (0, π/3]
(C) [π/3, π/2] (D) [π/6, π/3]

Q.2 If f(x) = $\frac{1}{x+1}$ and g (x) = $\frac{1}{\sqrt{x}-1}$, then common domain of function is -
(A) {x | x < 1, x ∈ R}
(B) {x | x ≥ 0, x ≠ 1, x ∈ R}
(C) {1}
(D) {–1}

Q.3 If f(x) = $\left(\frac{x}{1-|x|}\right)^{1/12}$, x ∈ R then domain of the function f(x) is -
(A) (–1, 0] (B) (–∞, –1) ∪ [0, 1)
(C) (–1, ∞) – {1} (D) None of these

Q.4 If f : R→ R, f(x) = tan x, then pre-image of –1 under f is-
(A) $\left\{n\pi - \frac{\pi}{4} \middle| n \in I\right\}$ (B) $\left\{n\pi + \frac{\pi}{4} \middle| n \in I\right\}$
(C) {nπ| n ∈ I } (D) None of these

Q.5 The domain of
f(x) = $\sqrt{[\cos(\sin x)]}$ + (1 – x)$^{-1}$ + $\sin^{-1}\left(\frac{x^2+1}{2x}\right)$
equal to -
(A) R – {1} (B) {–1}
(C) (1, ∞) (D) None of these

Q.6 If f : R → R, f(x) = x^3 + 3, and g : R → R, g(x) = 2x + 1, then f^{-1}og^{-1}(23) equals-
(A) 2 (B) 3
(C) (14)$^{1/3}$ (D) (15)$^{1/3}$

Q.7 The period of f(x) = $\frac{|\sin x| + |\cos x|}{|\sin x - \cos x|}$ is -
(A) π/2 (B) π
(C) 2π (D) None of these

Q.8 The function f(x) = $\frac{\sec^{-1} x}{\sqrt{x-[x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to -
(A) R
(B) R – {(–1, 1) ∪ {n : n ∈ Z}}
(C) R$^+$ – (0, 1)
(D) R$^+$ – {n : n ∈ N}

Q.9 The interval for which $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}$ holds-
(A) [0, ∞) (B) [0, 3]
(C) [0, 1] (D) [0, 2]

Q.10 The function
f(x) = $\cos^{-1}\left(\frac{|x|-3}{2}\right)$ + [log$_e$ (4 – x)]$^{-1}$ is defined for -
(A) [–1, 0] ∪ [1, 5]
(B) [–5, –1] ∪ [1, 4]
(C) [–5, –1] ∪ ([1, 4) – {3})
(D) [1, 4] – {3}

Q.11 Function f : R → R$^+$, f(x) = x^2 + 2 & g : R$^+$ → R, g(x) = $\left(1 - \frac{1}{1-x}\right)$ then the value of gof (2) is -
(A) 5/6 (B) 8/7
(C) 1/6 (D) 6/5

Q.12 Period of function $2^{\{x\}}$ + sin πx + $3^{\{x/2\}}$ + cos 2πx is (where { } represent fractional part of x)
(A) 2 (B) 1
(C) 3 (D) None of these

Q.13 Let f : (4, 6) → (6, 8) be a function defined by f(x) = x + [x/2] where [ ] represent G.I.F. then f^{-1}(x) is equal to -
(A) x – 2 (B) x – [x/2]
(C) – x – 2 (D) None of these

Q.14 The range of

$$f(x) = \sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{1-\cos x \ldots\ldots \infty}}}$$

is

(A) [0, 1] (B) [0, 1/2]

(C) [0, 2] (D) None of these

Q.15 Period of the function $f(x) = |\sin \pi x| + e^{3(x-[x])}$ (where [ ] represent G.I.F.) is -

(A) 1 (B) 2

(C) 1/3 (D) None of these

Q.16 If the domain of function $f(x) = x^2 - 6x + 7$ is $(-\infty, \infty)$, then the range of function is -

(A) $(-\infty, \infty)$ (B) $[-2, \infty)$

(C) $(-2, 3)$ (D) $(-\infty, -2)$

Q.17 Period of $f(x) = \sin 3\pi \{x\} + \tan \pi [x]$ where [ ] and {} represent of G.I.F and fractional part of x

(A) 1 (B) 2 (C) 3 (D) π

Q.18 If S be the set of all triangles and $f : S \rightarrow R^+$, $f(\Delta)$ = Area of Δ, then f is -

(A) one-one onto (B) one-one into

(C) many-one onto (D) many-one into

Q.19 Let $f(x) = \dfrac{9^x}{9^x + 3}$ & $f(x) + f(1-x) = 1$ then find value of $f\left(\dfrac{1}{1996}\right) + \left(\dfrac{2}{1996}\right) + \ldots\ldots + f\left(\dfrac{1995}{1996}\right)$ is -

(A) 998 (B) 997 (C) 997.5 (D) 998.5

Q.20 If period of $\dfrac{\cos(\sin nx)}{\tan(x/n)}$ $(n \in N)$ is 6π then n is equal to -

(A) 3 (B) 2 (C) 6 (D) 1

Q.21 If [x] and {x} represent the integral and fractional part of x respectively then value of $\sum_{r=1}^{2000} \dfrac{\{x+r\}}{2000}$ is

(A) x (B) [x]

(C) {x} (D) x + 2001

Q.22 The period of $f(x) = \cos(\sin x) + \cos(\cos x)$ is -

(A) $\pi/3$ (B) $\pi/6$

(C) π (D) $\pi/2$

Q.23 If f be the greatest integer function and g be the modulus function, then

$(gof)\left(-\dfrac{5}{3}\right) - (fog)\left(-\dfrac{5}{3}\right) =$

(A) 1 (B) –1

(C) 2 (D) 4

Q.24 The domain of function $f(x) = \log|\log x|$ is-

(A) $(0, \infty)$

(B) $(1, \infty)$

(C) $(0, 1) \cup (1, \infty)$

(D) $(-\infty, 1)$

Q.25 Domain of the function $\tan^{-1} x + \cos^{-1} x^2$ is -

(A) R– [–1, 1] (B) R– (–1, 1)

(C) (–1, 1) (D) [–1, 1]

Q.26 Which of the following functions are equal?

(A) $f(x) = x$, $g(x) = \sqrt{x^2}$

(B) $f(x) = \log x^2$, $g(x) = 2 \log x$

(C) $f(x) = 1$, $g(x) = \sin^2 x + \cos^2 x$

(D) $f(x) = x/x$, $g(x) = 1$

Q.27 The range of the function $y = \log_3 (5 + 4x - x^2)$ is -

(A) (0, 2] (B) $(-\infty, 2]$

(C) (0, 9] (D) None of these

Q.28 The range of the function $f(x) = |x-1| + |x-2|, -1 \le x \le 3$ is

(A) [1, 3] (B) [1, 5]

(C) [3, 5] (D) None of these

Q.29 The domain of the function

$$f(x) = \sin^{-1}\left(\frac{2-|x|}{4}\right) + \cos^{-1}\left(\frac{2-|x|}{4}\right) + \tan^{-1}\left(\frac{2-|x|}{4}\right)$$

is given by

(A) [– 3, 3] (B) [–6, 6]

(C) [0, 6] (D) None of these

Q.30 The domain of function

$f(x) = \dfrac{1}{\log_{10}(3-x)} + \sqrt{x+2}$ is -

(A) [–2, 3) (B) [–2, 3) – {2}

(C) [–3, 2] (D) [–2, 3] – {2}

Q.31 Domain of the function $f(x) = \dfrac{x-3}{(x-1)\sqrt{x^2-4}}$ is -

(A) (1, 2) (B) $(-\infty, -2) \cup (2, \infty)$

(C) $(-\infty,-2) \cup (1, \infty)$ (D) $(-\infty,\infty) - \{1, \pm 2\}$

Q.32 Domain of $\sin\left(\log\left(\dfrac{\sqrt{4-x^2}}{1-x}\right)\right)$ is -

(A) [–2, 1) (B) (–2, 1)

(C) (–2, 3] (D) None of these

Q.33 Let $f : R \to R$ be a function defined by $f(x) = x + \sqrt{x^2}$, then f is-

(A) injective (B) surjective

(C) bijective (D) None of these

Q.34 If $f(x) = e^{3x}$ and $g(x) = \ell n\, x$, x > 0, then (fog) (x) is equal to-

(A) 3x (B) x^3

(C) log 3x (D) 3 log x

Q.35 $f(x) = \log\left(\sqrt{x-3} + \sqrt{5-x}\right)$, $x \in R$ then domain of f(x) is

(A) [3, 5] (B) $[-\infty, 3] \cup [5, \infty]$

(C) {3, 5} (D) None of these

Q.36 Let $f : R \to R$ defined by $f(x) = \dfrac{\sin([x]\pi)}{x^2+2x+4}$, [.] = G.I.F., then which one is not true -

(A) f is periodic (B) f is even

(C) f is many-one (D) f is onto

Q.37 The domain of function

f(x) = log (3x – 1) + 2 log (x + 1) is -

(A) $[1/3, \infty)$ (B) [–1, 1/3]

(C) (–1, 1/3) (D) None of these

Q.38 If $f(x) = \dfrac{x}{\sqrt{1+x^2}}$, then (fofof) (x) is equal to-

(A) $\dfrac{3x}{\sqrt{1+x^2}}$ (B) $\dfrac{x}{\sqrt{1+3x^2}}$

(C) $\dfrac{3x}{\sqrt{1-x^2}}$ (D) None of these

Q.39 If f (x) be a polynomial satisfying f (x). f(1/x)= f (x) + f (1/x) and f(4) = 65 then f(6) = ?

(A) 176 (B) 217

(C) 289 (D) None of these

Q.40 If $f(x) = x^3 - x$ and g(x) = sin 2x, then-

(A) g [f(1)] = 1 (B) f (g (π/12)) = – 3/8

(C) g {f(2)} = sin 2 (D) None of these

Q.41 $f : R \to R$ is defined by $f(x) = \cos^2 x + \sin^4 x$ for $x \in R$ then the range of f (x) is -

(A) (3/4, 1) (B) [3/4, 1)

(C) [3/4, 1] (D) (3/4, 1)

Q.42 The natural domain of the real valued function defined by

$f(x) = \sqrt{x^2-1} + \sqrt{x^2+1}$ is-

(A) $1 < x < \infty$ (B) $-\infty < x < \infty$

(C) $-\infty < x < -1$ (D) $(-\infty, \infty) - (-1, 1)$

Q.43 If $f(x) = \dfrac{\sqrt{9-x^2}}{\sin^{-1}(3-x)}$, then domain of f is -

(A) [2, 3] (B) [2, 3)

(C) (2, 3] (D) None of these

Q.44 If f is a real function satisfying the relation f(x + y) = f (x) f(y) for all x, y $\in$ R and f(1) = 2, then a $\in$ N, for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, is given by -

(A) 2 (B) 4

(C) 3 (D) None of these

Q.45 Let f(x) = $\sqrt{(2+x-x^2)}$ and g(x) = $\sqrt{-x}+\frac{1}{\sqrt{x+2}}$. Then domain of f + g is given by -

(A) (–2 0] (B) [0, 1]

(C) [–1, 0] (D) (0, 1)

Q.46 Let f(x) = $\frac{x-[x]}{1-[x]+x}$, then range of f(x) is ([.] = G.I.F.) -

(A) [0, 1] (B) [0, 1/2]

(C) [1/2, 1] (D) [0, 1/2)

Q.47 If x = $\log_a bc$, y = $\log_b ca$, and z = $\log_c ab$, then $\frac{1}{1+x}+\frac{1}{1+y}+\frac{1}{1+z}$ equals-

(A) 1 (B) x + y + z

(C) abc (D) ab + bc + ca

Q.48 The range of 5 cos x – 12 sin x + 7 is-

(A) [–6, 20] (B) [–3, 18]

(C) [–6, 15] (D) None of these

Q.49 The domain of the function $\log_2\log_3\log_4(x)$ is-

(A) (1, ∞) (B) (2, ∞)

(C) (3, ∞) (D) (4, ∞)

NUMERIC RESPONSE TYPES QUESTIONS :

Q.50 If $f : R \to R$, $f(x) = 2x^3 - 7$, then find pre image of 9 under f.

Q.51 Find the period of the function satisfying the relation f(x) + f(x + 3) = 0 ∀ x ∈ R.

Q.52 If f(x) is a function satisfying f(x + y) = f(x) f(y) for all x, y ∈ N such that f(1) = 3 and $\sum_{x=1}^{n} f(x) = 120$, then find the value of n.

Q.53 Find the number of integers lying in the domain of the function

f (x) = $\sqrt{\log_{(0.5)}\left(\frac{5-2x}{x}\right)}$.

Q.54 If the range of the function f (x) = $6^x + 3^x + 6^{-x} + 3^{-x} + 2$ is [k, ∞) then find k.

Q.55 Find period of function $2^{\{x\}} + \sin \pi x + 3^{\{x/2\}} + \cos 2\pi x$ is (where { } represent fractional part of x)

Q.56 If period of $\frac{\cos(\sin nx)}{\tan(x/n)}$ (n ∈ N) is 6π then find value of n.

Q.57 If f : R → R satisfies f(x + y) = f(x) + f(y), for all x, y ∈ R and f(1) = 7, and $\sum_{r=1}^{n} f(r)$ = k (n)(n + 1) then find k.

Q.58 Let f (x) = $\frac{9^x}{9^x+3}$ and f(x) + f (1 – x) = 1 then find value of $f\left(\frac{1}{1996}\right)+f\left(\frac{2}{1996}\right)+ \ldots\ldots\ldots \ldots + f\left(\frac{1995}{1996}\right)$.

Q.59 If f (x) = $\begin{cases} 2, & x \in Q \\ -1, & x \notin Q \end{cases}$ then find fof(e).

EXERCISE -3

Q.1 The domain of definition of

$f(x) = \sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)} \times \frac{1}{x^2-36}$ is –

(A) $(x : x < 0, x \neq -6\}$
(B) $(x : x > 0, x \neq 1, x \neq 6\}$
(C) $(x : x > 1, x \neq 6\}$
(D) $(x : x \geq 1, x \neq 6\}$

Q.2 The function $f : R \to R$ defined by $f(x) = (x-1)(x-2)(x-3)$ is -
(A) one-one but not onto
(B) onto but not one-one
(C) both one and onto
(D) neither one-one nor onto

Q.3 The domain of f(x) is (0, 1) therefore domain of $f(e^x) + f(\ell n\,|x|)$ is -
(A) (–1, e) (B) (1, e)
(C) (–e, – 1) (D) (– e, 1)

Q.4 If $g : [-2, 2] \to R$ where $f(x) = x^3 + \tan x + \left[\frac{x^2+1}{p}\right]$ is a odd function then the value of p where [ ] represent G.I.F. -
(A) $-5 < p < 5$ (B) $p < 5$
(C) $p > 5$ (D) None of these

Q.5 If the graph of the function $f(x) = \frac{a^x - 1}{x^n(a^x+1)}$ is symmetrical about axis then n equals to –
(A) 2 (B) 2/3
(C) 1/4 (D) –1/3

Q.6 Let $f : R \to R$ be a function defined by $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$. Then -
(A) f is a bijection
(B) f is an injection only
(C) f is a surjection only
(D) f is neither an injection nor a surjection

Q.7 The value of $n \in I$ for which the function $f(x) = \frac{\sin nx}{\sin\left(\frac{x}{n}\right)}$ has 4π as its period is-
(A) 2 (B) 3 (C) 4 (D) 5

Q.8 If f(x) is an odd periodic function with period 2, then f(4) equals to-
(A) 0 (B) 2 (C) 4 (D) –4

Q.9 Domain of the function $f(x) = \sin^{-1}\left(\log_5 \frac{x^2}{5}\right)$ is-
(A) $[-5, -1] \cup [1, 5]$
(B) $[-5, 5]$
(C) $(-5, -1) \cup (1, 5)$
(D) None of these

Q.10 Domain of $f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$ is -
(A) $R - [-2, 2]$
(B) $R - [-1, 1]$
(C) $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$
(D) None of these

Q.11 If $f(x) = 3\sin\sqrt{\frac{\pi^2}{16} - x^2}$, then values of f(x) lie in
(A) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (B) [–2, 2]
(C) $\left[0, \frac{3}{\sqrt{2}}\right]$ (D) None of these

Q.12 The period of $f(x) = \sin\frac{x}{n!} + \cos\frac{x}{(n+1)!}$ is-
(A) non-periodic
(B) periodic with period $(2\pi)\,n!$
(C) periodic with period $2\pi\,(n+1)!$
(D) periodic with period $2\,(n+1)\,\pi$

Q.13 The function f(x) = max. [1 – x, 1+ x, 2] ; $x \in R$ is equivalent to -

(A) $f(x) = \begin{cases} 1-x, x \le -1 \\ 2, -1 < x < 1 \\ 1+x, x \ge 1 \end{cases}$

(B) $f(x) = \begin{cases} 1+x, x \le -1 \\ 2, -1 < x < 1 \\ 1-x, x \ge 1 \end{cases}$

(C) $f(x) = \begin{cases} 1-x, x \le -1 \\ 1, -1 < x < 1 \\ 1+x, x \ge 1 \end{cases}$

(D) None of these

Q.14 The domain of the function $f(x) = {}^{9-x}P_{x-5}$ is-
(A) [5, 7] (B) {5, 6, 7}
(C) {3, 4, 5, 6, 7} (D) None of these

Q.15 The range of the function $f(x) = {}^{9-x}P_{x-5}$ is -
(A) {1, 2, 3} (B) [1, 2]
(C) {1, 2, 3, 4, 5} (D) None of these

Q.16 Domain of the function

$f(x) = \log_2\left(-\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$ is -

(A) (0, 1) (B) (0, 1]
(C) [1, ∞) (D) (1, ∞)

Q.17 The period of f(x) = [sin 5x] + |cos 6x| is, where [.] is G.I.F -

(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) $\frac{2\pi}{5}$

Q.18 Period of $f(x) = \sin x + \tan\frac{x}{2} + \sin\frac{x}{2^2} + \tan\frac{x}{2^3} + ... + \sin\frac{x}{2^{n-1}} + \tan\frac{x}{2^n}$ is -

(A) π (B) 2π (C) $2^n\pi$ (D) $\frac{\pi}{2^n}$

Q.19 The period of

$f(x) = [x] + [2x] + ... + [nx] - \frac{n(n+1)}{2}x$ where

$n \in N$ and [ ] represent G.I.F. is -
(A) n (B) 1
(C) $\frac{1}{n}$ (D) None of these

Q.20 The function f : [–1/2, 1/2] → [–π/2, π/2] defined by $f(x) = \sin^{-1}(3x - 4x^3)$ is -
(A) both one-one and onto
(B) neither one-one nor onto
(C) onto but not one-one
(D) one-one but not onto

Q.21 The function f satisfies the equation

$3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$ for all real $x \ne 1$. The value of f(7) is -
(A) 8 (B) 4 (C) –8 (D) 11

Q.22 The domain of the function
$f(x) = \log_{3+x}(x^2 - 1)$ is -
(A) (–3, –1) ∪ (1, ∞)
(B) [–3, –1) ∪ [1, ∞)
(C) (–3, –2) ∪ (–2, –1) ∪ (1, ∞)
(D) [–3, –2) ∪ (–2, –1) ∪ [1, ∞)

Q.23 If x and y satisfy the equation y = 2[x] + 3 and y = 3[x – 2] simultaneously, then [x + y] is –
(A) 30 (B) 20 (C) 10 (D) None

Q.24 If $[\sin^{-1} x] > [\cos^{-1} x]$, where [.] denotes the greatest integer function, then complete set of values of x is -
(A) [cos 1, 1] (B) [sin 1, 1]
(C) [cos 1, sin 1] (D) [0, 1]

Q.25 $f(x) = \frac{x}{\sqrt{\sin(\ln x) - \cos(\ln x)}}$

(A) $(e^{\pi/4}, e^{5\pi/3})$ (B) $(e^{\pi/4}, e^{5\pi/4})$
(C) $(e^{-\pi/4}, e^{\pi/4})$ (D) None

EXERCISE -4

Old Examination Questions [AIEEE/JEE Main]

Q.1 Which of the following is not a periodic function - **[AIEEE 2002]**

(A) $\sin 2x + \cos x$ (B) $\cos \sqrt{x}$

(C) $\tan 4x$ (D) $\log \cos 2x$

Q.2 The period of $\sin^2 x$ is- **[AIEEE 2002]**

(A) $\pi/2$ (B) π (C) $3\pi/2$ (D) 2π

Q.3 The function $f : R \to R$ defined by $f(x) = \sin x$ is- **[AIEEE-2002]**

(A) into (B) onto

(C) one-one (D) many-one

Q.4 The range of the function $f(x) = \dfrac{2+x}{2-x}$, $x \neq 2$ is - **[AIEEE-2002]**

(A) R (B) R – {–1}

(C) R – {1} (D) R – {2}

Q.5 The function $f(x) = \log (x + \sqrt{x^2+1})$, is- **[AIEEE 2003]**

(A) neither an even nor an odd function

(B) an even function

(C) an odd function

(D) a periodic function

Q.6 Domain of definition of the function $f(x) = \dfrac{3}{4-x^2} + \log_{10}(x^3 - x)$, is- **[AIEEE 2003]**

(A) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(B) $(1, 2)$

(C) $(-1, 0) \cup (1, 2)$

(D) $(1, 2) \cup (2, \infty)$

Q.7 If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is- **[AIEEE 2003]**

(A) $\dfrac{7n(n+1)}{2}$ (B) $\dfrac{7n}{2}$

(C) $\dfrac{7(n+1)}{2}$ (D) $7n(n+1)$

Q.8 A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \dfrac{n-1}{2}, when\ n\ is\ odd \\ -\dfrac{n}{2}, when\ n\ is\ even \end{cases}$$ is

[AIEEE 2003]

(A) neither one-one nor onto

(B) one-one but not onto

(C) onto but not one-one

(D) one-one and onto both

Q.9 The range of the function $f(x) = {}^{7-x}P_{x-3}$ is- **[AIEEE 2004]**

(A) {1, 2, 3} (B) {1, 2, 3, 4, 5, 6}

(C) {1, 2, 3, 4} (D) {1, 2, 3, 4, 5}

Q.10 If $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3} \cos x + 1$, is onto, then the interval of S is- **[AIEEE 2004]**

(A) [0, 3] (B) [–1, 1]

(C) [0, 1] (D) [–1, 3]

Q.11 The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then- **[AIEEE 2004]**

(A) $f(x+2) = f(x-2)$

(B) $f(2+x) = f(2-x)$

(C) $f(x) = f(-x)$

(D) $f(x) = -f(-x)$

Q.12 The domain of the function $f(x) = \dfrac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is- **[AIEEE 2004]**

(A) [2, 3] (B) [2, 3)

(C) [1, 2] (D) [1, 2)

Q.13 Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1} \dfrac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval – **[AIEEE-2005]**

(A) $\left(0, \dfrac{\pi}{2}\right)$ (B) $\left[0, \dfrac{\pi}{2}\right)$

(C) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ (D) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

Q.14 A real valued function f(x) satisfies the functional equation f(x – y) = f(x) f(y) – f (a – x) f(a + y) where a is a given constant and f(0) = 1, then f(2a – x) is equal to – **[AIEEE-2005]**

(A) –f(x) (B) f(x)
(C) f(a) + f(a – x) (D) f(–x)

Q.15 The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function f(x) = $4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right)$ + log (cos x) defined, is- **[AIEEE 2007]**

(A) [0, π] (B) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
(C) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (D) $\left[0, \frac{\pi}{2}\right)$

Q.16 Let f : N → Y be a function defined as f(x) = 4x + 3 where Y = |y ∈ N : y = 4x + 3 for some x ∈ N|. inverse of f is – **[AIEEE 2008]**

(A) g(y) = $4 + \frac{y+3}{4}$ (B) g(y) = $\frac{y+3}{4}$
(C) g(y) = $\frac{y-3}{4}$ (D) g(y) = $\frac{3y+4}{3}$

Q.17 For real x, let f(x) = x^3 + 5x + 1, then – **[AIEEE 2009]**

(A) f is one - one but not onto R
(B) f is onto R but not one - one
(C) f is one - one and onto on R
(D) f is neither one - one nor onto R

Q.18 Let $f(x) = (x + 1)^2 - 1$, $x \geq -1$

Statement - 1 :
The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.
Statement - 2 :
f is a bijection. **[AIEEE 2009]**

(A) Statement -1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement -1
(B) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement -1.
(C) Statement -1 is true, Statement-2 is false.
(D) Statement -1 is false, Statement-2 is true.

Q.19 The domain of the function

$f(x) = \frac{1}{\sqrt{|x| - x}}$ is : **[AIEEE 2011]**

(A) $(-\infty, \infty)$ (B) $(0, \infty)$
(C) $(-\infty, 0)$ (D) $(-\infty, \infty) - \{0\}$

Q.20 Let A and B be nonempty set in R and f : A → B be a bijective function.
Statement-1: f is an onto function
Statement-2 : There exists a function g : B → A such that fog = I_B.
[AIEEE Online- 2012]

(A) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true

Q.21 The range of the function f(x) = $\frac{x}{1+|x|}$, x ∈ R is : **[AIEEE Online- 2012]**

(A) [–1, 1] (B) R
(C) R – {0} (D) (–1, 1)

Q.22 If P(S) denotes the set of all subsets of a given set S, then the number of one to one functions from the set S = {1, 2, 3} to the set P(S) is : **[AIEEE Online- 2012]**

(A) 24 (B) 8
(C) 336 (D) 320

Q.23 Let A = {1, 2, 3, 4} and R : A → A be the relation defined by R={(1, 1), (2, 3), (3, 4), (4, 2)}. The correct statements is : **[JEE Main Online -2013]**

(A) R does not have an inverse
(B) R is not a one to one function
(C) R is an onto function
(D) R is not a function

Q.24 If $a \in R$ and the equation $-3(x-[x])^2 + 2(x-[x]) + a^2 = 0$ (where [x] denotes the greatest integer $\le x$) has no integral solution, then all possible values of a lie in the interval : **[JEE Main - 2014]**

(A) $(-\infty, -2) \cup (2, \infty)$ (B) $(-1, 0) \cup (0, 1)$

(C) (1, 2) (D) (–2, –1)

Q.25 Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals : **[JEE Main - 2014]**

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Q.26 If $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$, $x \in R$, then the equation f(x) = 0 has : **[JEE Main Online -2014]**

(A) no solution

(B) one solution

(C) two solution

(D) more than two solutions

Q.27 Let f be an odd function defined on the set of real numbers such that for $x \ge 0$,
$f(x) = 3 \sin x + 4 \cos x$.
Then $f(x)$ at $x = -\frac{11\pi}{6}$ is equal to : **[JEE Main Online -2014]**

(A) $\frac{3}{2} + 2\sqrt{3}$ (B) $-\frac{3}{2} + 2\sqrt{3}$

(C) $\frac{3}{2} - 2\sqrt{3}$ (D) $-\frac{3}{2} - 2\sqrt{3}$

Q.28 Let $f : R \to R$ be defined by $f(x) = \frac{|x|-1}{|x|+1}$ then f is **[JEE Main Online -2014]**

(A) Both one – one and onto

(B) One – one but not onto

(C) Onto but not one – one

(D) Neither one – one nor onto

Q.29 Let $f(n) = \left[\frac{1}{3} + \frac{3n}{100}\right]n$, where [n] denotes the greatest integer less than or equal to n. Then $\sum_{n=1}^{56} {}_1 f(n)$ is equal to :

[JEE Main Online -2014]

(A) 56 (B) 689 (C) 1287 (D) 1399

Q.30 The function $f(x) = |\sin 4x| + |\cos 2x|$, is a periodic function with period. **[JEE Main Online -2014]**

(A) 2π (B) π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Q.31 If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \ne 0$, and $S = \{x \in R : f(x) = f(-x)\}$; then S : **[JEE Main -2016]**

(A) is an empty set

(B) contains exactly one element

(C) contains exactly two elements

(D) contains more than two elements

Q.32 For $x \in \mathbf{R}$, $x \ne 0$, $x \ne 1$, let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x))$, n = 0, 1, 2, ... Then the value of $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$ is equal to -

[JEE Main Online -2016]

(A) $\frac{8}{3}$ (B) $\frac{5}{3}$ (C) $\frac{4}{3}$ (D) $\frac{1}{3}$

Q.33 The function $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is **[JEE Main -2017]**

(A) injective but not surjective

(B) surjective but not injective

(C) neither injective nor surjective

(D) invertible

Q.34 Let $f(x) = 2^{10}x + 1$ and $g(x) = 3^{10}x - 1$. If $(fog)(x) = x$, then x is equal to -

[JEE Main Online -2017]

(A) $\frac{2^{10}-1}{2^{10}-3^{-10}}$ (B) $\frac{1-2^{-10}}{3^{10}-2^{-10}}$

(C) $\frac{3^{10}-1}{3^{10}-2^{-10}}$ (D) $\frac{1-3^{-10}}{2^{10}-3^{-10}}$

Q.35 The function $f : N \to N$ defined by $f(x) = x - 5\left[\frac{x}{5}\right]$, where N is the set of natural numbers and [x] denotes the greatest integer less than or equal to x, is -

[JEE Main Online -2017]

(A) one-one but not onto

(B) one-one and onto

(C) neither one-one nor onto

(D) onto but not one-one

Q.36 Let $f : A \to B$ be a function defined as $f(x) = \frac{x-1}{x-2}$, where $A = R - \{2\}$ and $B = R - \{1\}$. Then f is -

[JEE-Main Online-2018]

(A) invertible and $f^{-1}(y) = \frac{2y+1}{y-1}$

(B) invertible and $f^{-1}(y) = \frac{3y-1}{y-1}$

(C) no invertible

(D) invertible and $f^{-1}(y) = \frac{2y-1}{y-1}$

Q.37 The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is :

[JEE Main - 2019]

(A) $(1, 2) \cup (2, \infty)$

(B) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

(C) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(D) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

Q.38 Let $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{ x \in R : f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true? **[JEE Main - 2019]**

(A) $g(f(S)) \neq S$ (B) $f(g(S)) = S$

(C) $f(g(S)) \neq f(S)$ (D) $g(f(S)) = g(S)$

Q.39 For $x \in (0, 3/2)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$ If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to : **[JEE Main - 2019]**

(A) $\tan\frac{7\pi}{12}$ (B) $\tan\frac{11\pi}{12}$

(C) $\tan\frac{\pi}{12}$ (D) $\tan\frac{5\pi}{12}$

Q.40 If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to: **[JEE Main - 2020]**

(A) $-\frac{3}{2}$ (B) $-\frac{1}{2}$

(C) $\frac{3}{2}$ (D) $\frac{1}{2}$

Q.41 Let $f : R \to R$ be a function which satisfies $f(x + y) = f(x) + f(y) \forall x, y \in R$. If $f(1) = 2$ and $g(n) = \sum_{k=1}^{(n-1)} f(k), n \in N$ then the value of n, for which $g(n) = 20$, is : **[JEE Main - 2020]**

(A) 5 (B) 9 (C) 20 (D) 4

Q.42 Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x-1}$. Then the composition function $f(g(x))$ is : **[JEE Main - 2021]**

(A) onto but not one-one

(B) both one-one and onto

(C) one-one but not onto

(D) neither one-one nor onto

Q.43 Let A = {2, 3, 4, 5, ... , 30} and '$\simeq$' be an equivalence relation on A × A, defined by (a, b) $\simeq$ (c, d), if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to : **[JEE Main - 2021]**
(A) 5 (B) 6 (C) 8 (D) 7

Q.44 Let f : R – {3} → R – {1} be defined by $f(x)=\frac{x-2}{x-3}$. Let g : R → R be given as g(x) = 2x – 3. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x)=\frac{13}{2}$ is equal to **[JEE Main - 2021]**
(A) 7 (B) 2 (C) 5 (D) 3

Q.45 Let ***f*** : **N** → **R** be a function such that $f(x + y) = 2f(x)\, f(y)$ for natural numbers x and y. If f(A) = 2, then the value of α for which $\sum_{k=1}^{10} f(\alpha+k)=\frac{512}{3}(2^{20}-1)$ holds, is : **[JEE Main-2022]**
(A) 2 (B) 3 (C) 4 (D) 6

Q.46 Let f(x) = $\frac{x-1}{x+1}$, x ∈ R – {0, – 1, 1}. If $f^{n+1}(x) = f(f^{n}(x))$ for all n ∈ N, then $f^{6}(6) = f^{7}(7)$ is equal to **[JEE Main-2022]**
(A) $\frac{7}{6}$ (B) $-\frac{3}{2}$ (C) $\frac{7}{12}$ (D) $-\frac{11}{12}$

Q.47 Let : f : R → R be defined as f(x) = x – 1 and g : R – {1, – 1} → R be defined as
g(x) = $\frac{x^2}{x^2-1}$
Then the function fog is: **[JEE Main-2022]**
(A) One-one but not onto
(B) Onto but not one-one
(C) Both one-one and onto
(D) Neither one-one nor onto

Q.48 Let a function f : N → N be defined by

$$f(n)=\begin{cases} 2n, & n=2,4,6,8.... \\ n-1, & n=3,7,11,15.... \\ \frac{n+1}{2} & n=1,5,9,13.... \end{cases}$$

then, f is **[JEE Main-2022]**
(A) One-one but not onto
(B) Onto but not one-one
(C) Neither one-one nor onto
(D) One-one and onto

Q.49 Let f : **R** → **R** be a continuous function such that f(3x) – f(x) = x. If f(8) = 7, then f(14) is equal to **[JEE Main-2022]**
(A) 4 (B) 10 (C) 11 (D) 16

Q.50 Let f(x) and g(x) be two real polynomials of degree 2 and 1 respectively. If f(g(x)) = $8x^2 - 2x$ and g(f(x)) = $4x^2 + 6x + 1$, then the value of f(B) + g(B) is___________. **[JEE Main-2022]**

Q.51 The number of functions f, from the set A = {x ∈ N : $x^2 - 10x + 9 \le 0$} to the set B = {n^2 : n ∈ N} such that f(x) ≤ $(x - 3)^2 + 1$, for every x ∈ A, is ____ . **[JEE Main-2022]**

Q.52 If $f(x)=\frac{2^{2x}}{2^{2x}+2}$, x ∈ R , then $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+.......+f\left(\frac{2022}{2023}\right)$ is equal to **[JEE Main-2023]**
(A) 1010 (B) 2011 (C) 1011 (D) 2010

Q.53 Let $f(x)$ be a function such that $f(x + y) = f(x)\cdot f(y)$ for all x, y ∈ N. If $f(1)$ = 3 and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is **[JEE Main-2023]**
(A) 8 (B) 9 (C) 6 (D) 7

Q.54 Let f, g and h be the real valued functions defined of R as

$$f(x)\begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 1, & x=0 \end{cases}, \quad g(x)\begin{cases} \frac{\sin(x+1)}{(x+1)}, & x \neq -1 \\ 1, & x=-1 \end{cases}$$

and $h(x) = 2[x] - f(x)$, where [x] is the greatest integer ≤ x.
Then the value of $\lim_{x\to 1} g(h(x-1))$ is **[JEE Main-2023]**
(A) 1 (B) –1 (C) sin(1) (D) 0

Q.55 Let $f : (0, 1) \to R$ be a function defined by $f(x) = \frac{1}{1-e^{-x}}$, and $g(x) = (f(-x) - f(x))$.
Consider two statements
(I) g is an increasing function in (0, 1)
(II) g is one-one in (0, 1)
Then, **[JEE Main-2023]**
(A) Only (I) is true
(B) Both (I) and (II) are true
(C) Only (II) is true
(D) Neither (I) nor (II) is true

Q.56 Let $f : R - \{0, 1\} \to$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. then f(2) is equal to : **[JEE Main-2023]**
(A) $\frac{9}{2}$ (B) $\frac{9}{4}$ (C) $\frac{7}{3}$ (D) $\frac{7}{4}$

Q.57 The function $f : N - \{1\} \to N$; defined by f(n) = the highest prime factor of n, is: **[JEE Main-2024]**
(A) Both one-one and onto
(B) Neither one-one nor onto
(C) One-one only
(D) Onto only

Q.58 Let $f : R - \left\{\frac{-1}{2}\right\} \to R$ and $g : R - \left\{\frac{-5}{2}\right\} \to R$ be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$. Then, the domain of the function fog is: **[JEE Main-2024]**
(A) R (B) $R - \left\{\frac{-5}{2}\right\}$
(C) $R - \left\{\frac{-7}{4}\right\}$ (D) $R - \left\{\frac{-5}{2}, -\frac{7}{4}\right\}$

Q.59 If $f(x) = \begin{cases} 2+2x, & -1 \le x < 0 \\ 1-\frac{x}{3}, & 0 \le x \le 3 \end{cases}$;
$g(x) = \begin{cases} -x, & -3 \le x \le 0 \\ x, & 0 < x \le 1 \end{cases}$,
then range of (fog)(x) is- **[JEE Main-2024]**
(A) [0, 1] (B) (0, 1] (C) [0, 1) (D) [0, 3)

Q.60 Let the sum of the maximum and the minimum values of the function $f(x) = \frac{2x^2-3x+8}{2x^2+3x+8}$ be $\frac{m}{n}$ where gcd(m, n) = 1. Then m + n is equal to : **[JEE Main-2024]**
(A) 182 (B) 217 (C) 195 (D) 201

Q.61 Let $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$ be a real valued function. If α and β are respectively the minimum and the maximum values of f, then $\alpha^2 + 2\beta^2$ is equal to **[JEE Main-2024]**
(A) 44 (B) 42 (C) 24 (D) 38

Q.62 Let A = {1, 2, 3, 4} and B = {1, 4, 9, 16}. Then the number of many-one functions $f: A \to B$ such that $1 \in f(A)$ is equal to : **[JEE Main-2025]**
(A) 127 (B) 151 (C) 163 (D) 139

Q.63 Let $f(x) = \log_e x$ and $g(x) = \frac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}$. Then the domain of fog is- **[JEE Main-2025]**
(A) R (B) $(0, \infty)$ (C) $[0, \infty)$ (D) $[1, \infty)$

Q.64 If the domain of the function
$f(x) = \log_7(1 - \log_4(x^2 - 9x + 18))$ is $(\alpha, \beta) \cup (\gamma, \delta)$, then $\alpha + \beta + \gamma + \delta$ is equal to **[JEE Main-2025]**
(A) 17 (B) 18 (C) 15 (D) 16

Q.65 If the range of the function $f(x) = \frac{5-x}{x^2-3x+2}$, $x \ne 1, 2$, is $(-\infty. \alpha] \cup [\beta, \infty)$, then $\alpha^2 + \beta^2$ is equal to : **[JEE Main-2025]**
(A) 194 (B) 192 (C) 188 (D) 190

Q.66 Number of functions $f : \{1, 2, \ldots, 100\} \to \{0, 1\}$, that assign 1 to exactly one of the positive integers less than or equal to 98, is equal to ______. **[JEE Main-2025]**

EXERCISE -5

Old Examination Questions [IIT JEE Advanced]

Q.1 If function $f(x) = \frac{1}{2} - \tan\left(\frac{\pi x}{2}\right)$; $(-1 < x < 1)$ and $g(x) = \sqrt{3 + 4x - 4x^2}$, then the domain of gof is – **[IIT-1990]**

(A) $(-1, 1)$ (B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\left[-1, \frac{1}{2}\right]$ (D) $\left[-\frac{1}{2}, -1\right]$

Q.2 If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, where $[x]$ stands for the greatest integer function, then **[IIT- 1991]**

(A) $f\left(\frac{\pi}{2}\right) = -1$ (B) $f(\pi) = 1$

(C) $f\left(\frac{\pi}{4}\right) = 2$ (D) None of these

Q.3 The value of b and c for which the identity $f(x + 1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$, are- **[IIT- 1992]**

(A) $b = 2, c = 1$ (B) $b = 4, c = -1$

(C) $b = -1, c = 4$ (D) None

Q.4 Let $f(x) = \sin x$ and $g(x) = \ell n\,|x|$. If the ranges of the composite functions fog and gof are R_1 and R_2 respectively, then – **[IIT- 1994]**

(A) $R_1 = \{u : -1 < u < 1\}$,
$R_2 = \{v : -\infty < v < 0\}$

(B) $R_1 = \{u : -\infty < u < 0\}$,
$R_2 = \{v : -1 < v < 1\}$

(C) $R_1 = \{u : -1 < u < 2\}$,
$R_2 = \{v : -\infty < v < 0\}$

(D) $R_1 = \{u : -1 \le u \le 1\}$,
$R_2 = \{v : -\infty < v \le 0\}$

Q.5 Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval **[IIT- 1994]**

(A) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (B) $\left(-1, \frac{5\pi}{6}\right)$

(C) $(-1, 2)$ (D) $\left(\frac{\pi}{6}, 2\right)$

Q.6 Let $f(x) = (x + 1)^2 - 1$, $(x \ge -1)$. Then the set $S = \{x : f(x) = f^{-1}(x)\}$ is - **[IIT- 1995]**

(A) Empty

(B) $\{0, -1\}$

(C) $\{0, 1, -1\}$

(D) $\left\{0, -1, \frac{-3 + i\sqrt{3}}{2}, \frac{-3 - i\sqrt{3}}{2}\right\}$

Q.7 If $f(1) = 1$ and $f(n + 1) = 2f(n) + 1$ if $n \ge 1$, then $f(n)$ is- **[IIT- 1995]**

(A) 2^{n+1} (B) 2^n

(C) $2^n - 1$ (D) $2^{n-1} - 1$

Q.8 If f is an even function defined on the interval $(-5, 5)$, then the real values of x satisfying the equation $f(x) = f\left(\frac{x+1}{x+2}\right)$ are **[IIT- 1996]**

(A) $\frac{-1 \pm \sqrt{5}}{2}, \frac{-3 \pm \sqrt{5}}{2}$

(B) $\frac{-1 \pm \sqrt{3}}{2}, \frac{-3 \pm \sqrt{3}}{2}$

(C) $\frac{-2 \pm \sqrt{5}}{2}$

(D) None of these

Q.9 Let $f(x) = [x]\sin\left(\frac{\pi}{[x+1]}\right)$, where [.] denotes the greatest integer function. The domain of f is .... **[IIT 1996]**

(A) $\{x \in R\,|\, x \in [-1, 0)\}$

(B) $\{x \in R\,|\, x \notin [1, 0)\}$

(C) $\{x \in R\,|\, x \notin [-1, 0)\}$

(D) None of these

Q.10 If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$ & $g\left(\frac{5}{4}\right) = 1$, then $(gof)(x) =$ **[IIT 1996]**

(A) -2 (B) -1 (C) 2 (D) 1

Q.11 If g(f(x)) = |sin x| and f(g(x)) = $(\sin\sqrt{x})^2$, then **[IIT 1998]**

(A) f(x) = $\sin^2 x$, g(x) = $\sqrt{x}$

(B) f(x) = sin x, g(x) = |x|

(C) f(x) = x^2, g(x) = $\sin\sqrt{x}$

(D) f and g cannot be determined

Q.12 If f(x) = 3x – 5, then $f^{-1}(x)$ **[IIT 1998]**

(A) is given by $\frac{1}{3x-5}$

(B) is given by $\frac{x+5}{3}$

(C) does not exist because f is not one-one

(D) does not exist because f is not onto

Q.13 If the function f : [1, ∞) → [1, ∞) is defined by f(x) = $2^{x(x-1)}$, then f^{-1} (x) is **[IIT 1999]**

(A) $\left(\frac{1}{2}\right)^{x(x-1)}$

(B) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

(C) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$

(D) not defined

Q.14 The domain of definition of the function y(x) given by the equation $2^x + 2^y = 2$ is – **[IIT Scr. 2000]**

(A) 0 < x < 1 (B) 0 < x < 1

(C) –∞ < x < 0 (D) –∞ < x < 1

Q.15 Let f(θ) = sinθ (sinθ + sin 3θ), then f(θ) **[IIT 2000]**

(A) ≥ 0 only when θ ≥ 0

(B) ≤ 0 for all θ

(C) ≥ 0 for all real θ

(D) ≤ 0 only when θ ≤ 0

Q.16 The number of solutions of $\log_4(x-1) = \log_2(x-3)$ is – **[IIT Scr. 2001]**

(A) 3 (B) 1 (C) 2 (D) 0

Q.17 Let f(x) = $\frac{\alpha x}{x+1}$, x ≠ – 1, then for what value of α, f{f(x)} = x. **[IIT Scr. 2001]**

(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) 1 (D) –1

Q.18 The domain of definition of f (x) = $\frac{\log_2(x+3)}{x^2+3x+2}$ is **[IIT Scr. 2001]**

(A) R / {–2, –2}

(B) (– 2, ∞)

(C) R/ {–1, –2, –3}

(D) (–3, ∞) / {–1, –2}

Q.19 If f : [1, ∞) → [2, ∞) is given by f(x) = x + $\frac{1}{x}$ then f^{-1} (x) equals – **[IIT Scr. 2001]**

(A) $\frac{x+\sqrt{x^2-4}}{2}$ (B) $\frac{x}{1+x^2}$

(C) $\frac{x-\sqrt{x^2-4}}{2}$ (D) $1+\sqrt{x^2-4}$

Q.20 Let g(x) = 1 + x – [x] and

f(x) = $\begin{cases} -1 & ; \; x<0 \\ 0 & ; \; x=0 \\ 1 & ; \; x>0 \end{cases}$. Then for all x, f(g(x)) is equal to : (where [.] denotes the greatest integer function): **[IIT Scr. 2001]**

(A) x (B) 1 (C) f(x) (D) g(x)

Q.21 Suppose f(x) = $(x + 1)^2$ for x ≥ – 1. If g(x) is the function whose graph is the reflection of the graph of f(x) with respect to the line y = x, then g(x) equals - **[IIT Scr. 2002]**

(A) $-\sqrt{x}-1$, x ≥ 0 (B) $\frac{1}{(x+1)^2}$, x > – 1

(C) $\sqrt{x+1}$, x ≥ – 1 (D) $\sqrt{x}-1$, x ≥ 0

Q.22 Let function f : R → R be defined by f(x) = 2x + sin x for x ∈ R. Then f is - **[IIT Scr. 2002]**

(A) one to one and onto

(B) one to one but NOT onto

(C) onto but NOT one to one

(D) neither one to one nor onto

Q.23 Let f(x) = $\frac{x}{1+x}$ defined as [0, ∞) → [0, ∞), f(x) is– **[IIT Scr.2003]**

(A) one-one & onto

(B) one-one but not onto

(C) not one-one but onto

(D) neither one-one nor onto

Q.24 Find the range of f(x) = $\frac{x^2+x+2}{x^2+x+1}$ is–

[IIT Scr.2003]

(A) $(1, \infty)$ (B) $\left(1, \frac{11}{7}\right)$

(C) $\left(1, \frac{7}{3}\right]$ (D) $\left(1, \frac{7}{5}\right)$

Q.25 Domain of f(x) = $\sqrt{\sin^{-1}(2x)+\pi/6}$ is–

[IIT Scr.2003]

(A) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\left[-\frac{1}{4}, \frac{1}{4}\right]$ (D) $\left[-\frac{1}{2}, \frac{1}{4}\right]$

Q.26 Let f(x) = sin x + cos x & g(x) = x^2 — 1, then g(f (x)) will be invertible for the domain-

[IIT Scr.2004]

(A) $x \in [0, \pi]$ (B) $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(C) $x \in \left[0, \frac{\pi}{2}\right]$ (D) $x \in \left[-\frac{\pi}{2}, 0\right)$

Q.27 $f(x)=\begin{cases} x & x\in Q \\ 0 & x\notin Q\end{cases}$; $g(x)=\begin{cases} 0 & x\in Q \\ x & x\notin Q\end{cases}$

then (f – g) is **[IIT Scr.2005]**

(A) one-one, onto

(B) neither one-one, nor onto

(C) one-one but not onto

(D) onto but not one-one

Q.28 Let f(x) = x^2 and g (x) = sin x for all x ∈ R. Then the set of all x satisfying (fogogof) (x) = (gogof) (x), where (fog)(x) = f(g(x)), is - **[IIT 2011]**

(A) $\pm\sqrt{n\pi}$, n ∈ {0, 1, 2, ...}

(B) $\pm\sqrt{n\pi}$, n ∈ {1, 2, ...}

(C) $\frac{\pi}{2} + 2n\pi$, n ∈ {..., – 2, –1, 0, 1, 2, ...}

(D) $2n\pi$, n ∈ {..., – 2, –1, 0, 1, 2, ...}

Q.29 The function f : [0, 3] → [1, 29], defined by f(x) = $2x^3 - 15x^2 + 36x + 1$, is **[IIT 2012]**

(A) one-one and onto

(B) onto but not one-one.

(C) one-one but not onto

(D) neither one-one nor onto.

Q.30 Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}$ (β – α) is ____ . **[JEE - Advance 2018]**

Q.31 If the function f : R → R is defined by f(x) = |x| (x – sin x), then which of the following statements is **TRUE** ?

[IIT-Advance 2020]

(A) f is one-one, but **NOT** onto

(B) f is onto, but **NOT** one-one

(C) f is **BOTH** one-one and onto

(D) f is **NEITHER** one-one **NOR** onto

Q.32 Let f : [0, 2] → R be the function defined by f(x) = $(3 - \sin 2\pi x)) \sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right)$. If α, β ∈ [0, 2] are such that {x ∈ [0, 2] : f(x) ≥ 0} = [α, β], then the value of β – α is ______ **[IIT-Advance 2020]**

Q.33 Let the function f : [0, 1] → R be defined by $f(x) = \frac{4^x}{4^x+2}$. Then the value of

$f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + f\left(\frac{3}{40}\right) + \ldots\ldots + f\left(\frac{39}{40}\right) - f\left(\frac{1}{2}\right)$ is ______

[IIT-Advance 2020]

Q.34 Let $f:(0,1)\to\mathbf{R}$ be the function defined as $f(x)=\sqrt{n}$ if $x\in\left[\frac{1}{n+1},\frac{1}{n}\right)$ where $n\in\mathbf{N}$. Let $g:(0,1)\to\mathbf{R}$ be a function such that $\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}\,dt < g(x) < 2\sqrt{x}$ for all $x\in(0,1)$.

Then $\lim_{x\to 0} f(x)g(x)$ **[JEE - Advance 2023]**

(A) does NOT exist (B) is equal to 1
(C) is equal to 2 (D) is equal to 3

Q.35 Let $n\ge 2$ be a natural number and $f:[0,1]\to R$ be the function defined by

$$f(x)=\begin{cases} n(1-2nx) & \text{if } 0\le x\le \frac{1}{2n} \\ 2n(2nx-1) & \text{if } \frac{1}{2n}\le x\le \frac{3}{4n} \\ 4n(1-nx) & \text{if } \frac{3}{4n}\le x\le \frac{1}{n} \\ \frac{n}{n-1}(nx-1) & \text{if } \frac{1}{n}\le x\le 1 \end{cases}$$

If n is such that the area of the region bounded by the curves $x=0$, $x=1$, $y=0$ and $y=f(x)$ is 4, then the maximum value of the function f is **[JEE - Advance 2023]**

Q.36 Let $f:[1,\infty)\to R$ be a differentiable function such that $f(1)=\frac{1}{3}$ and $3\int_1^x f(t)dt = xf(x)-\frac{x^3}{3}, x\in[1,\infty)$. Let e denote the base of the natural logarithm. Then the value of f(e) is **[JEE - Advance 2023]**

(A) $\frac{e^2+4}{3}$ (B) $\frac{\log_e 4+e}{3}$

(C) $\frac{4e^2}{3}$ (D) $\frac{e^2-4}{3}$

Q.37 Let $f:R\to R$ be a function defined by

$$f(x)=\begin{cases} x^2\sin\left(\frac{\pi}{x^2}\right), & \text{if } x\ne 0, \\ 0, & \text{if } x=0 \end{cases}$$

Then which of the following statement is TRUE ? **[JEE - Advance 2024]**

(A) $f(x)=0$ has infinitely many solutions in the interval $\left[\frac{1}{10^{10}},\infty\right)$.

(B) $f(x)=0$ has no solutions in the interval $\left[\frac{1}{\pi},\infty\right)$.

(C) The set of solutions of $f(x)=0$ in the interval $\left(0,\frac{1}{10^{10}}\right)$ is finite.

(D) $f(x)=0$ has more than 25 solutions in the interval $\left(\frac{1}{\pi^2},\frac{1}{\pi}\right)$.

Q.38 Let R denote the set of all real numbers. Let $a_i, b_i\in R$ for $i\in\{1,2,3\}$.
Define the functions $f:R\to R$, $g:R\to R$, and $h:R\to R$ by

$f(x)=a_1+10x+a_2x^2+a_3x^3+x^4$,
$g(x)=b_1+3x+b_2x^2+b_3x^3+x^4$,
$h(x)=f(x+1)-g(x+2)$.

If $f(x)\ne g(x)$ for every $x\in R$, then the coefficient of x^3 in h(x) is

[JEE - Advanced 2025]

(A) 8 (B) 2
(C) – 4 (D) – 6

Q.39 Let R denote the set of all real numbers. Let $f:R\to R$ be a function such that $f(x)>0$ for all $x\in R$, and $f(x+y)=f(x)f(y)$ for all $x,y\in R$.
Let the real numbers $a_1, a_2, \ldots, a_{50}$ be in an arithmetic progression. If $f(a_{31})=64f(a_{25})$, and $\sum_{i=1}^{50} f(a_i)=3(2^{25}+1)$, then the value of $\sum_{i=6}^{30} f(a_i)$ is ________.

[JEE - Advanced 2025]

ANSWER KEY

EXERCISE - 1

1. (C)	**2.** (A)	**3.** (B)	**4.** (B)	**5.** (A)	**6.** (C)	**7.** (D)
8. (C)	**9.** (C)	**10.** (D)	**11.** (A)	**12.** (A)	**13.** (D)	**14.** (A)
15. (B)	**16.** (C)	**17.** (B)	**18.** (B)	**19.** (A)	**20.** (B)	**21.** (A)
22. (C)	**23.** (B)	**24.** (A)	**25.** (B)	**26.** (B)	**27.** (C)	**28.** (B)
29. (B)	**30.** (B)	**31.** (B)	**32.** (B)	**33.** (B)	**34.** (C)	**35.** (B)
36. (A)	**37.** (C)	**38.** (C)	**39.** (B)	**40.** (C)	**41.** (B)	**42.** (B)
43. (A)	**44.** (C)	**45.** (B)	**46.** (B)	**47.** (D)	**48.** (A)	**49.** (B)
50. (B)	**51.** (A)	**52.** (A)	**53.** (C)	**54.** (A)	**55.** (D)	**56.** (D)
57. (C)	**58.** (C)	**59.** (C)	**60.** (C)	**61.** (C)	**62.** (C)	**63.** (B)
64. (B)	**65.** (B)	**66.** (C)	**67.** (D)	**68.** (C)	**69.** (B)	**70.** (B)
71. (D)	**72.** (D)	**73.** (C)	**74.** (C)	**75.** (C)	**76.** (A)	**77.** (D)
78. (A)	**79.** (B)	**80.** (B)	**81.** (C)	**82.** (C)	**83.** (B)	**84.** (D)
85. (D)	**86.** (C)	**87.** (D)	**88.** (A)	**89.** (C)	**90.** (C)	**91.** (C)

EXERCISE -2

1. (C)	**2.** (B)	**3.** (B)	**4.** (A)	**5.** (B)	**6.** (A)	**7.** (B)
8. (B)	**9.** (C)	**10.** (C)	**11.** (D)	**12.** (A)	**13.** (A)	**14.** (C)
15. (A)	**16.** (B)	**17.** (A)	**18.** (C)	**19.** (C)	**20.** (C)	**21.** (C)
22. (D)	**23.** (A)	**24.** (C)	**25.** (D)	**26.** (C)	**27.** (B)	**28.** (B)
29. (B)	**30.** (B)	**31.** (B)	**32.** (B)	**33.** (D)	**34.** (B)	**35.** (A)
36. (D)	**37.** (D)	**38.** (B)	**39.** (B)	**40.** (B)	**41.** (C)	**42.** (D)
43. (B)	**44.** (C)	**45.** (C)	**46.** (D)	**47.** (A)	**48.** (A)	**49.** (D)
50. 2.00	**51.** 6.00	**52.** 4.00	**53.** 1.00	**54.** 6.00	**55.** 2.00	**56.** 6.00
57. 3.50	**58.** 997.50	**59.** 2.00				

EXERCISE -3

1. (C)	**2.** (B)	**3.** (C)	**4.** (C)	**5.** (D)	**6.** (D)	**7.** (A)
8. (A)	**9.** (A)	**10.** (C)	**11.** (C)	**12.** (C)	**13.** (A)	**14.** (B)
15. (A)	**16.** (A)	**17.** (C)	**18.** (C)	**19.** (B)	**20.** (A)	**21.** (B)
22. (C)	**23.** (A)	**24.** (B)	**25.** (B)			

EXERCISE -4

1. (B)	**2.** (B)	**3.** (A,D)	**4.** (B)	**5.** (C)	**6.** (A)	**7.** (A)
8. (D)	**9.** (A)	**10.** (D)	**11.** (B)	**12.** (B)	**13.** (D)	**14.** (A)
15. (D)	**16.** (C)	**17.** (C)	**18.** (B)	**19.** (C)	**20.** (A)	**21.** (D)
22. (C)	**23.** (C)	**24.** (B)	**25.** (A)	**26.** (B)	**27.** (C)	**28.** (D)
29. (D)	**30.** (C)	**31.** (C)	**32.** (B)	**33.** (B)	**34.** (B)	**35.** (C)
36. (D)	**37.** (C)	**38.** (D)	**39.** (B)	**40.** (B)	**41.** (A)	**42.** (C)
43. (D)	**44.** (C)	**45.** (C)	**46.** (B)	**47.** (D)	**48.** (D)	**49.** (B)
50. 18.00	**51.** 1440	**52.** (C)	**53.** (D)	**54.** (A)	**55.** (B)	**56.** (B)
57. (B)	**58.** (B)	**59.** (A)	**60.** (D)	**61.** (B)	**62.** (B)	**63.** (A)
64. (B)	**65.** (A)	**66.** (392)				

EXERCISE-5

1. (B)	**2.** (A)	**3.** (B)	**4.** (D)	**5.** (D)	**6.** (B)	**7.** (C)
8. (A)	**9.** (C)	**10.** (D)	**11.** (A)	**12.** (B)	**13.** (B)	**14.** (D)
15. (C)	**16.** (B)	**17.** (D)	**18.** (D)	**19.** (A)	**20.** (C)	**21.** (D)
22. (A)	**23.** (B)	**24.** (C)	**25.** (A)	**26.** (B)	**27.** (A)	**28.** (A)
29. (B)	**30.** 119.00	**31.** (C)	**32.** 1.00	**33.** 19.00	**34.** (C)	**35.** 8
36. (C)	**37.** (D)	**38.** (C)	**39.** 96.00			